

**ASYMMETRIC APODIZATION FOR THE COMMA ABERRATED POINT SPREAD FUNCTION**A.N.K. Reddy<sup>1</sup>, D.K. Sagar<sup>2</sup>, S.N. Khonina<sup>1,3</sup><sup>1</sup> Samara National Research University, Samara, Russia,<sup>2</sup> Optics Research Group, Department of Physics, University College of Science,  
Osmania University, Hyderabad, Telangana, India,<sup>3</sup> Image Processing Systems Institute – Branch of the Federal Scientific Research Centre  
“Crystallography and Photonics” of Russian Academy of Sciences, Samara, Russia**Abstract**

This paper deals with the study of light flux distributions in the point spread function formed by an optical system with a one-dimensional aperture under the influence of the coma aberration. The traditional design of an asymmetric optical filter improves the resolution of a diffraction-limited optical imaging system. In this approach we explore the control of monochromatic aberrations through pupil engineering with asymmetric apodization. This technique employs the amplitude and phase apodization for the mitigation of the effects of third-order aberrations on the diffracted image. On introducing the coma wave aberration effect, the central peak intensity in the field of diffraction is a function of the edge strips width and the amplitude apodization parameter of a one-dimensional pupil filter, whereas the magnitude of the reduction of optical side-lobes is a function of the degree of phase apodization at the periphery of the aperture. The analytically computed results are illustrated graphically in terms of point spread function curves under various considerations of the coma aberrations and a different degree of amplitude and phase apodization. Hence, for the optimum values of apodization, the axial resolution has been analyzed using well-defined quality criteria.

**Keywords:** asymmetric apodization, amplitude masking, optical filters, resolution, monochromatic aberrations.

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**1. Introduction**

From previous studies on apodization [1–4], it is the process of suppression of optical side-lobes in the diffraction field of an optical imaging system. Also, phase apodization is used for increasing depth of field [5–7]. The efficiency of apodization technique is always associated with the design of the pupil function. As reported in the earlier works [8–13], asymmetric apodization is the process of simultaneous suppression of optical side-lobes and sharpening the main peak of the point spread function (PSF). Vortex phase functions are also used to reduce the focal spot of high-aperture focusing system [14, 15]. There exist a significant number of investigations on the concept of asymmetric apodization, [16–20] pursue: suppression of side-lobes with asymmetric apodization. Asymmetric apodization applied to arrays of the circular aperture. Two-point resolution of asymmetrically apodised optical systems. Asymmetric apodization employs to achieve axial and lateral resolution in confocal scanning systems or optical imaging systems. Direct detection of the image of extra solar planets is technically possible by employing asymmetric apodization. All these studies are the basis for the current investigation. It is obvious that asymmetric apodization alters the distribution of light flux enclosed in the diffraction pattern and renders the resolution of an optical system, i.e. obtained suppressed side-lobes and steep principal maxima on one side of the PSF at the cost of enhanced side-lobes and broadened main peak on the other side of the PSF.

Aberrations usually limit the performance of optical systems. Except the case of sharply focusing when aberrations may reduce the focal spot size [21, 22], although usually at the cost of growth of side lobes. We have known that aberrations are produced by the deviations from the actual size, shape and positions of an image as calculated by simple equations. Point Spread Functions has been investigated for different aperture shapes and apodised filters in an optical system, degraded their imaging resolution primarily by defocusing and primary aberration effects. The third order off-axis aberration such as coma is important in designing optical systems in perspective of large field imaging. There are certain numbers of studies [23–28] are done on the images of point or line objects under the defocusing and third order aberrations. An optical system free from spherical aberration produces an image of point body is positioned off the axis, like comet shaped image. When a point object is situated far off the axis, a streak of light that appears to emanate from focusing spot at the periphery of the view filed. Coma is regularly considered as a most challenging aberration due to the irregularity in the image. It is also one of the easiest aberrations to demonstrate like spherical aberration. Coma aberration can become a severe problem if the optical imaging system is out of proper alignment. Reduction of coma is done with the help of suitable shaped apertures and materials of suitable apodization technique by placing it in a suitable position of an optical system. By looking into the deep mechanism of apodization process, we understood that a pupil function with a suitable ampli-

tude and phase apodization is very helpful to control the third order aberrations for achieving super resolution. For an instance, the reduction of coma, i.e. this restricts the outer zones and permits only the central zones to refract the light rays from point objects. This approach is very effective in enhancing the resolution of optical imaging systems over a limited region in the diffracted image by controlling third order monochromatic aberrations. In this paper, we designed one-dimensional shaded aperture functions divided into three zones. Two narrow edge strips apply phase apodization and the shade in the central zone applies amplitude apodization to modify a spatial distribution of light intensity in the image plane of systems. By employing asymmetric apodization we obtained the impulse response of the optical system under third order aberrations. The optical system taken in this paper is pretended to be linear, shift variant diffraction limited and asymmetric. The relation between the coded image and Point Spread Function is accomplished mathematically. The current study has an extensive number of applications viz. Astronomical observations, confocal imaging, medical imaging, optical communications and adaptive optics laser imaging, etc.

**2. Theory and Formulation**

A patch of light could be expressed as a distribution of light radiation in the image plane of optical systems. A diffracted image is a spatial pattern of light distribution. the resolution of the image is evaluated based on the end to which an image allows the detector to recognize the axial shape of the point spread function in the presence of wave aberration with the suitable phase and amplitude modulation in the pupil plane of the optical system. In the case of one-dimensional amplitude and phase apertures impulse response of optical systems is the Fourier transform of the pupil functions. Fig.1 shows the model of asymmetric optical imaging systems.

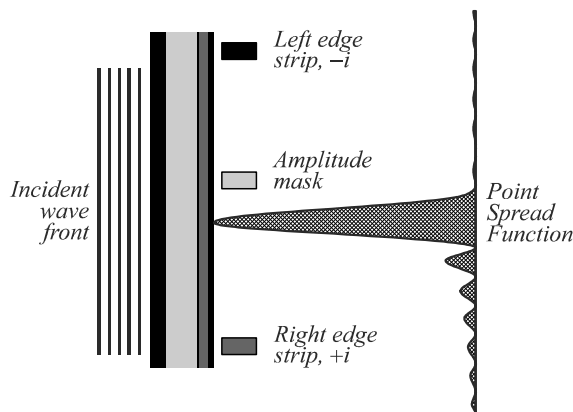


Fig. 1. General scheme of one dimensional optical imaging system

The aperture systems consist of two narrow edge strips of equal in width, that are opaque and transparent. Due to the deep suppression ability and constant operational angles throughout the periphery zones, they impose phase functions which are complex conjugated. The central zone of pupil functions is being opaque. In the presence of third order

erration such as coma the total diffraction amplitude transmittance  $V(u)$  into the image plane is equal to the sum of the individual phase transmittances contributing by periphery edge strips for which amplitude transmittance is unity and the amplitude transmittance contributing by the central zone of the pupil functions of width  $(1-2S)$  Hanning amplitude filter is given by:

$$\begin{cases} \text{Left opaque} & = e^{-i\pi/2}, -0.5 \leq \rho < -0.5 + S, \\ \text{Central mask} & -0.5 + S \leq \rho \leq 0.5 - S, \\ \text{Right opaque} & = e^{i\pi/2} \quad 0.5 - S < \rho \leq 0.5. \end{cases} \quad (1)$$

On the introducing the third order wave aberration coma, the amplitude impulse response of the optical system with one-dimensional amplitude and phase filter is:

$$\begin{aligned} V(u) = & \int_{-0.5}^{-0.5+S} -i \exp(i2u\rho) d\rho + \\ & \left\{ \uparrow \text{left edge strip zone} \uparrow \right\} \\ & \left\{ \leftrightarrow \text{central zone with amplitude mask,} \right. \\ & \left. \text{known as Hanning amplitude filter} \downarrow \right\} \\ & + \int_{-0.5+S}^{0.5-S} \cos(\pi\beta\rho) \exp\left[-i(\cos\pi\frac{Cr^3}{3})\right] \exp(i2u\rho) d\rho + \\ & \left\{ \downarrow \text{Right edge strip zone} \downarrow \right\} \\ & + \int_{0.5-S}^{0.5} i \exp(i2u\rho) d\rho. \end{aligned} \quad (2)$$

The resultant amplitude distribution of light  $V(u)$  in the focal region of one-dimensional optical imaging systems is equal to the sum of diffraction amplitudes of three zones, by taking the square modulus of  $V(u)$  gives the resultant intensity PSF  $P(u)$  which is the actual quantifiable quantity. Here  $u = (2\pi/\lambda) \sin \theta$ ,  $\rho$  is the coordinate in the aperture function, 'u' is the quantity has dimension of inverse length units,  $\lambda$  is the wavelength of monochromatic light radiation.  $C$  is the coma aberration control parameter. The width of the edge strip ( $S$ ) is a parameter determined from the minimum of the square of the intensity in the side-lobe range. In this case, the side-lobe region is stated by 'u', which is connected to the angle of orientation  $\theta$ .  $S$  is the control parameter of phase apodization or asymmetric apodization or width of the periphery strips. ' $\beta$ ' is the degree of amplitude apodization in the central region of the pupil function. The effects of the third order wave aberrations on the point spread function of optical systems are given, as a result:

$$P(u) = |V(u)|^2. \quad (3)$$

**3. Results**

In the present paper by employing one-dimensional pupil filters of amplitude and phase apodization, the PSF in the presence of third order wave aberrations such as the coma are tabulated and presented. An iterative Gauss quadrature method of numerical integration has been developed and applied to obtain the PSF from the equation

(3), which is a function of dimension less optical coordinate 'u' is varying from -25 to +25. This method holds important properties and accuracy to find the resolution and improved side-lobe suppression on the both sides of diffracted PSF under third order monochromatic aberration. However, results are computed only on one side of the diffracted image known as good side of the PSF. On the good side of the PSF has suppressed side-lobes and narrowed central peak. Here we investigated the lateral resolution of the PSF in terms of the central peak width or its FWHM and axial resolution of the PSF has calculated with respect to optical side-lobes intensities for different degree of amplitude - phase apodization and monochromatic aberration, coma. These parameters in addition to FWHM are enough for the resolution analysis of the asymmetric optical imaging system under various considerations.

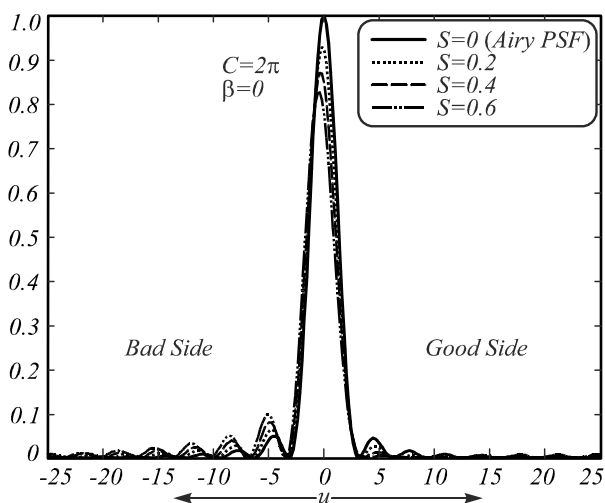


Fig. 2. PSF profile curves in the presence of high coma (C) aberration for various degrees of phase-only apodization ( $S \neq 0$  and  $\beta = 0$ )

Fig. 2 illustrates that the central peak on the left half pattern of the point spread function is shifted, broadened at the cost of suppressed side-lobes and steep central peak on the right half pattern. The magnitude of the central peak shift depends on the degree of phase apodization takes place at edge strips of the aperture system. We observed that the magnitude of suppression of optical side-lobes is increasing on the good side of the pattern with the edge strip width (S). It also found that for optimum value of phase apodization  $S = 0.06$ , higher order optical side-lobes on the good of the pattern are completely controlled and the first-order optical side-lobe is found with almost zero-level intensity. In this case the PSF is suffering from high coma aberration due to the result of variations in the refractive index by light waves passing through the various zones of the lens or aperture system. Primarily the effect of coma aberration due to the light waves passing through the periphery of aperture system can reach the image plane closer to the light waves emerging from the central region of the lens or aperture.

For  $S = 0.06$  and  $\beta = 0$  (phase-only apodization), there exists a maximum correction of coma aberration on one-side of the diffracted image, i.e. for optimum phase

apodization at periphery strips of the one-dimensional aperture, the distinct shape of the image on one side has corrected by reducing the edge ringing effect illuminated that a negligible amount of light intensity distribution into the optical side-lobes of the PSF of the image. In Fig. 2 a solid black line curve represents Airy Pattern for easy comparison with asymmetrically apodised PSF under the coma aberration. The magnitude of the effect observed in Fig. 2 increases with degree of amplitude apodization in the central region of the pupil function. It has shown in detail in Fig. 3 as a function of reduced optical coordinate 'u'. Fig. 3 depicts that, for all values of edge strips width and  $\beta = 0.5$  similar trend results are found like Fig. 2. But in this case axial resolution of the PSF is increasing more effectively relative to the values of S at definite amplitude apodization. It is found that for all values of S lower and higher order optical side-lobes are nearly vanished on the good side of the PSF. It is also observed that the suppression of optical side-lobes on the bad side of the PSF improved considerably from Fig. 2 to Fig. 3 with the degree of amplitude apodization  $\beta$ . In this case it is found that for  $S = 0.06$  and  $\beta = 0.5$ , the magnitude of correction of the coma aberration increases to a larger extent on the good side of the PSF however similar observations are noticed in Fig. 2 in the presence of a transparent central region ( $\beta = 0$ ). The study found in the Fig. 3 facilitates to detect the direct image of faint object in the close vicinity of the bright point object, known as two-point resolution of optical systems under optical aberrations.

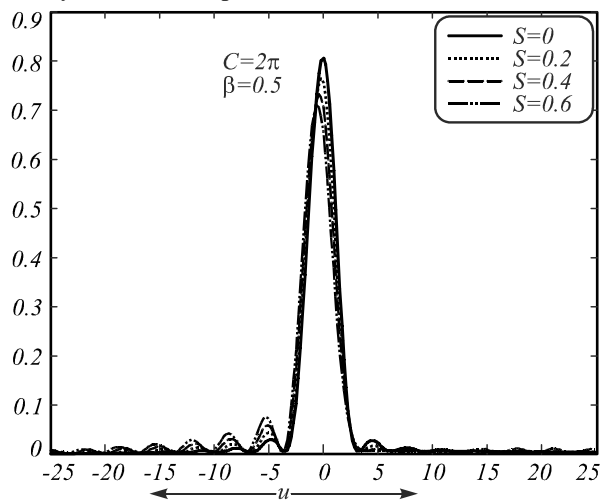


Fig. 3. PSF profile curves in the presence of high coma aberration for various degrees of phase apodization (S) when the central region of the pupil function is amplitude apodised ( $\beta = 0.5$ )

The effect of the Hanning-amplitude filter on the suppression of optical side-lobes for optimum asymmetric apodization underneath coma has been depicted in Fig. 4. It shows that the asymmetry in the light distribution of the PSF does not vary with the values of amplitude apodization control parameter  $\beta$  but the suppression of side-lobes have a great dependence on the values of  $\beta$ . It displays a proof for agreeable outcome of the amplitude apodization mask in the central region of the pupil function. By employing amplitude and phase apodization the correction of

coma aberration on the good side is much appreciable than earlier cases. Listed PSF curves in the Fig. 4 also reveal that in the presence of coma aberration the side-lobes suppression considerably improved on the bad side of the PSF to the degree of amplitude apodization at a definite periphery strip width  $S$ . Here the intensity of optical side-lobes have obtained on the bad side of the PSF is less than zero apodization case for the central region ( $\beta=0$ ) of the aperture. for  $S=0.06$  and  $\beta=0.75$ , the side-lobes are suppressed on the good side and the intensity of the side-lobes found on the bad side, which are less than non-apodized ones. It renders the axial resolution on both sides of the PSF. For  $\beta=0.6$ , the side-lobes on the good side of the PSF are suppressed to the zero intensity level, whereas on the bad side, the intensity of the side-lobes is not less than the case of  $\beta=0.75$ . Hence, it concludes that on both sides of the PSF the optimum values of  $S$  and  $\beta$  are 0.06 and 0.75, respectively. For  $S>0.06$  the intensity of the side-lobes on the good side of the PSF increased more than unapodized ones and also the PSF found with non-zero minima on the good side, results the resolution is degraded so that the study is limited to the certain values of ' $S$ '. In this study, coma has reduced to an appreciable extent by the use of suitable shaped complex apodization mask placed at a measurable distance from the image plane. This mask restricts the outer zones on one side of the field and permits only the central zones to refract the light rays at the cost of a considerable amount of blurred image on the other side of the field.

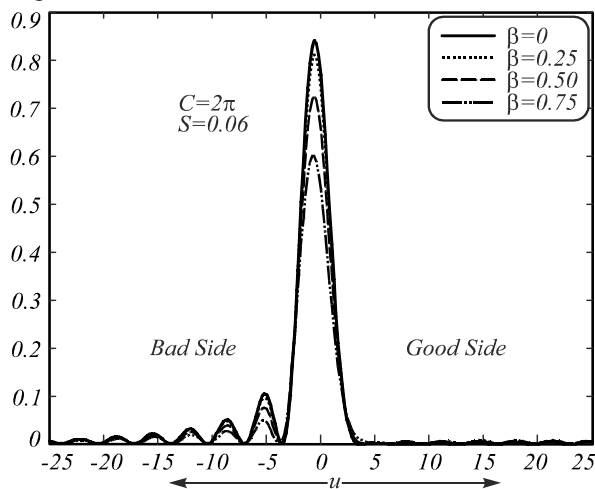


Fig. 4. PSF Profile curves for various values of amplitude apodization parameter  $\beta$  under the influence of high coma aberration for optimum asymmetric apodization

Table 1. FWHM values of Apodised PSF under the Wave aberration, Coma

$S$	$\beta=0$ $C=0$	$\beta=0.5$ $C=0$	$\beta=0$ $C=\pi/2$	$\beta=0.5$ $C=\pi/2$	$\beta=0$ $C=2\pi$	$\beta=0.5$ $C=2\pi$
0	2.7831	2.9824	2.7831	2.92137	2.7838	2.9222
0.06	2.9972	3.1180	3.0048	3.07021	2.9733	3.03618

Table 1 shows FWHM values of PSF for various values of  $S$ ,  $\beta$  and  $C$  (coma aberration). From the computed values in Table.1 for  $S=0$  and  $\beta=0.5$ , as coma aberration increases from 0 to  $2\pi$  the FWHM decreases from 2.982 to 2.921 whereas for  $S=0.06$  and  $\beta=0$ , as coma

aberration increases to  $2\pi$  the FWHM increases from 2.997 to 3 and then decreases to 2.973. But for  $S=0.06$  and  $\beta=0.5$ , the FWHM decreases from 3.118 to 3.036 as wave aberration effect increase from 0 to  $2\pi$ . It is clear that in the presence of Hanning amplitude apodization only there is significant improvement in the axial resolution of the coma effected PSF. On the other hand, for optimum anti-phase apodization the PSF under the high coma effect obtains FWHM values which are lower than aberration-free case. It concludes that by employing optimum asymmetric apodization the lateral resolution of the PSF under coma aberrations has been improved. Fig. 5 illustrates the distribution of light radiation into the image plane under different considerations and high coma aberration in which solid black line represents Airy PSF. It is observed that for aberration made PSF subjected to apodization, optical side-lobes are found to very lower than the Airy case, whereas central peak is narrowed on the good side of the pattern.

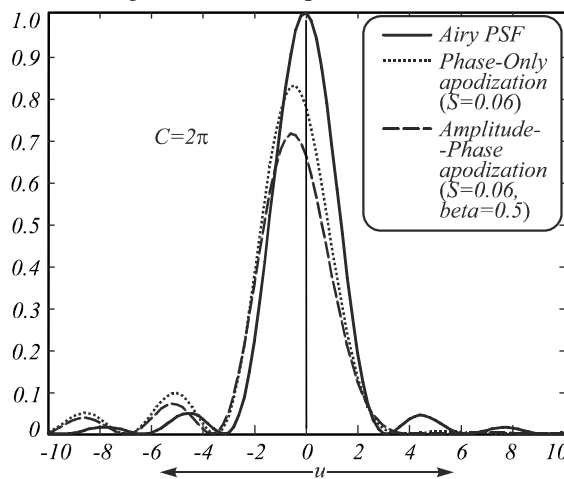


Fig. 5. Comma effected Point Spread Function under different considerations

#### 4. Conclusion

Asymmetric PSF under the third wave aberration effect such as coma is formulated and investigated. The resolution of the point spread function under various considerations has been studied by employing amplitude-phase apodization. It is concluded that for  $S=0.06$  and  $\beta=0.5$ , the magnitude of correction of coma increases to a larger extent on the good side of the PSF and lower, higher order optical side-lobes are vanished to zero intensity level. This effect can give rise to super-resolution for two closely spaced companions with widely varying in their intensities. In the presence of high coma aberration the suppression of optical side-lobes have been improved on the bad side of the PSF also to the degree of amplitude apodization  $\beta$  at a definite periphery strip width  $S=0.06$ . It emphasized that for the modified PSF on both sides, the optimum values of  $S$  and  $\beta$  are 0.06 and 0.75, respectively. For clear central region aperture by employing optimum asymmetric apodization at the edges of the aperture we found the PSF with decreased FWHM under high coma effect. The design of this complex apodization mask is technically feasible by superimposition of metal-

lic and dielectric films on glass substrate using vacuum thermal evaporation thin film deposition systems and for future prospects, it is very helpful for high contrast astronomical imaging and spectroscopy.

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