

# [1] An algorithm for designing a DOE to form optical traps of a preset configuration

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## Abstract

We proposed an algorithm for calculating the DOE forming vortex beams with a preset distribution of minima and maxima. We reviewed several versions of this algorithm. We presented the calculation of some configurations of traps to capture transparent and non-transparent micro-objects.

**Keywords:** LIGHT TRAP, ITERATIVE ALGORITHM, VORTEX AXICON.

**Citation:** SKIDANOV R.V. AN ALGORITHM FOR DESIGNING A DOE TO FORM OPTICAL TRAPS OF A PRESET CONFIGURATION / SKIDANOV R.V., GANCHEVSKAYA S.V. // COMPUTER OPTICS. – 2015. – VOL. 39(2). – P. 181-186

## Introduction

There are more papers now increasingly appear dedicated to optical micromanipulation, including the increasing interest to manipulating with large groups of micro-objects [1-5]. If the task of forming a large number of the light traps for transparent microobjects is well reported [1-5], the formation of a preset configuration of the light traps for non-transparent microobjects is rather limitedly represented [6]. Since the calculation of diffractive optical elements (DOE) for forming the preset intensity distribution is considered to be an incorrect problem, we may often need to use the iterative methods of calculation. There are several approaches to optimization of the diffractive optical elements using direct search methods. First of all, it concerns the exhaustive algorithm [7] when the search is carried out throughout the whole region of permissible solutions without any limitations. However, searching a proper solution will require considerable time and large computational costs, being sometimes almost impossible for state-of-the-art IT equipment.

A faster computational method is the organization of the iterative algorithm constructed either on minimizing the residual function or on a genetic approach. It is also possible to arrange the iterative calculation of diffractive optical elements based on standard optimization algorithms, i.e. the coordinatewise descent, the gradient descent, etc.

Optimization of the phase function of the diffractive optical element focusing laser emission on a longitudinal segment in a framework of a ray-optical approach was considered in papers [8-10]. Let's note that the ray-optical calculation method doesn't allow the diffraction effects to be taken into consideration. Iterative methods

of calculation of the DOE [11], based on the Gerchberg-Saxton algorithm [12] seem to be known, too. There are also the exact iterative methods of calculation based on the electromagnetic approach. Paper [14] presents an optimization procedure for binary diffractive optical elements destined for the formation of the "light bottle" distributions. Disadvantage of this algorithm is that it is possible to form only one trap of this type. Papers [15], [16] present the stochastic optimization algorithms of the quantized DOE. In paper [15] preset longitudinal intensity distributions have been developed, and in paper [16] the calculation algorithm of the DOE with the quantized phase function has been analyzed. Paper [17] presents a hybrid algorithm which combines advantages of the genetic algorithm and the local search method. Calculation of intensity distributions for the light traps has its own specific characteristics, i.e. the distribution itself is not so much important as its major parts (minima, maxima). In this context the iterative algorithm may be implemented based on the discrete parameters such as, for example, topological charges included in the superposition of the light fields.

### 1. The coordinatewise descent algorithm with minimizing the mean-square deviation

Paper [18] introduces the method for the formation of the light fields, which are just the superposition of a combination of several vortex light beams generated by vortex axicons. Nevertheless, as shown in paper [18], the resulting distributions represent a set of regularly situated light minima and maxima. Based on the proposed approach, we can form the light traps of completely different types: from Bessel beams [19] to linear traps [20].

It entitles to believe that for any arbitrary preset arrangement of the light traps we can choose such combination of topological charges of vortex beams included in the superposition, which shall form the required configuration of intensity minima and maxima.

To begin with, we shall make an attempt to develop the exact search algorithm of the phase function for the formation of the given intensity distribution. We shall choose the coordinatewise descent algorithm as a basis for the calculation algorithm. We propose to use topological charges of vortex beams included in the superposition as the coordinates of this algorithm. As the residual function we used the mean-square raster deviation  $\delta$  of the developed light field.

$$\delta = \sqrt{\frac{\sum_{i,j=0}^N (I_{ij} - I_{ij}^{Et})^2}{N^2}}, \quad (1)$$

where  $N$  is the image dimension,  $I_{ij}$  is the intensity distribution in a generated image,  $I_{ij}^{Et}$  is the intensity distribution in a master image. In this case a preliminary normalization procedure was carried out for master and generated images.

As a starting point of the algorithm we used the DOE forming the superposition of several vortex beams with similar topological charges equal to 1. In order to separate the beams, we divided adjacent zones in the DOE introducing the phase displacement by  $\pi$  [5]. Then we determined a direction of variation of the topological charge of the first zone, i.e. whether the topological charge is to be increased or decreased. Then the topological charge of the zone was changed by  $\pm 1$  at each step depending on the selected direction. Calculation of the intensity distribution was carried out by Fourier transformation of the function of the DOE complex transmission. The mean-square deviation was calculated at each step, and this sequence was performed until the value of the mean-square deviation began to increase. Once this is done, exactly the same sequence of steps was carried out in the next zone of the DOE. A total amount of zones of the DOE was a variable parameter of the algorithm and it changed from 3 to 8 depending on the task. The following two types of images were used as a standard: the image with an exact solution, i.e. as the standard we used the light field, which was the superposition of several vortex beams, and the arbitrary intensity distribution with several light traps. For standard intensity distributions, which have been formed as intensity distributions under diffraction on vortex axicon, i.e. having exact solutions as follows from this search, the algorithm often found a proper vortex axicon (Fig. 1a c), or found the axicon forming the intensity distribution which was close enough to the standard (Fig. 1b d).

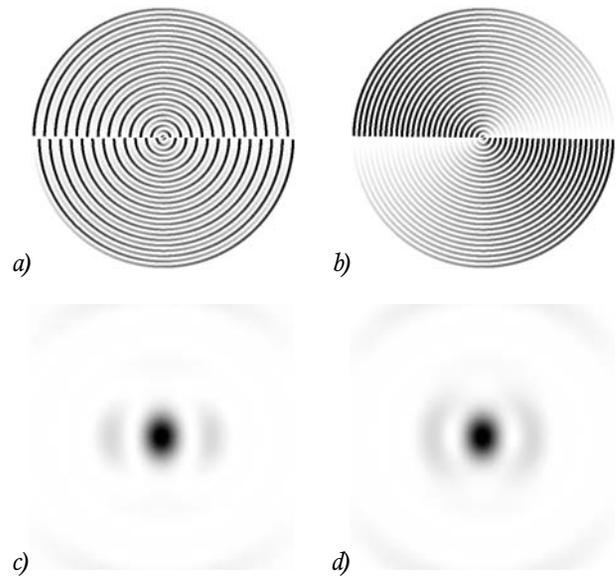


Fig. 1. The phase function of the DOE (a) forming the intensity distribution in the form of a single trap (b) in full correspondence to the standard, the phase function of the DOE (b) forming the intensity distribution in the form of the single trap (b) with incomplete correspondence to the standard

Fig. 1 shows the search results among the elements which have four separate zones with topological charges. As can be seen from Fig. 1, the above algorithm comes either to the exact solution (Fig.1c) or to the solution which is very close to the exact solution (Fig.1d). Fig. 2 shows the sequence of intensity distributions obtained in the course of subsequent iterations of the algorithm.

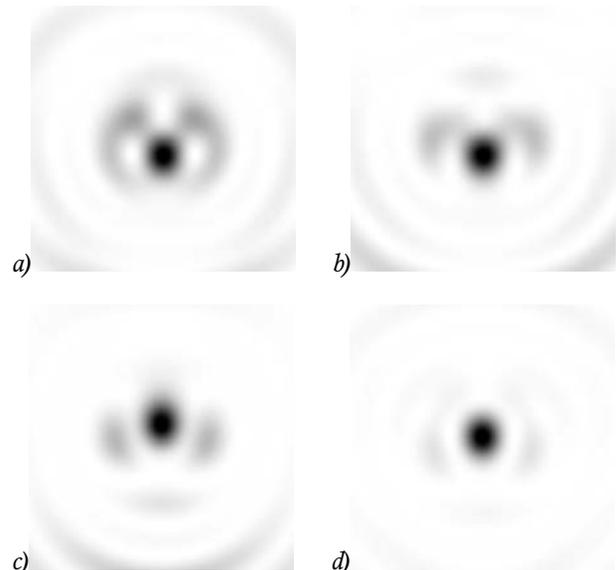


Fig. 2. Exact solution search stages

Unfortunately, in most cases it turns out that for the standard with no exact solution the algorithm does not come to a proper solution (Fig. 3).

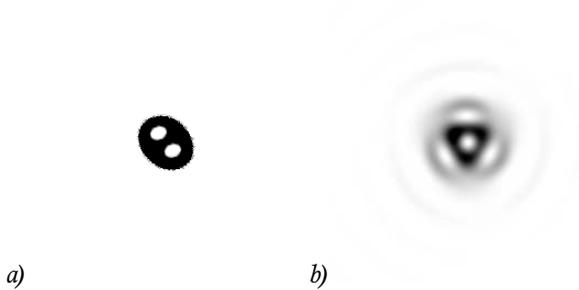


Fig. 3. The standard intensity distribution (a), the intensity distribution found by the algorithm

As can be seen from Fig. 3, the resulted image is very much far off in its distribution from the standard. At the same time a quite considerable part of the energy of the obtained intensity distribution is located outside the area defined on the standard and, in general, particularly due to these areas with relatively low intensity, the algorithm obtains a low mean-square deviation, and it comes to an incorrect solution. In order to remedy this, the algorithm has been modified.

**2. The coordinatewise descent algorithm with minimizing the residual function based on the combination of the mean-square deviation and a first order moment of the intensity distribution by r**

The residual function has been changed in the new algorithm. Instead of the mean-square deviation we used, as the residual function, the linear combination of the mean-square deviation, and the first order moment of the intensity distribution by r:

$$\delta_1 = k_1\delta + k_2m, \tag{2}$$

where the weight factors  $k_1, k_2$  were empirically selected values ( $k_1=0,8, k_2=0,2$ ), and m was determined as follows:

$$m = \frac{\int_0^R rI(r) dr}{\int_0^R I(r) dr} \tag{3}$$

where  $r = \sqrt{x^2 + y^2}$  is one of the polar coordinates in an image plane. However, it should be understood that the standard intensity distribu-

tion is to be located in the center of the rasterized image.

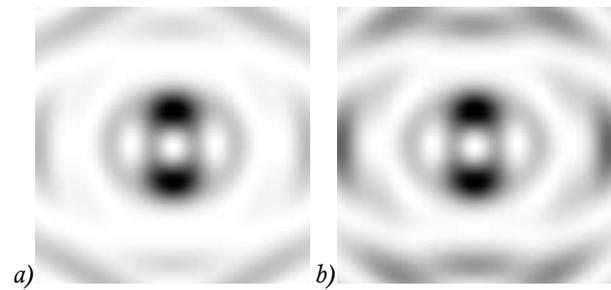


Fig. 4. Successful search of the standard distribution (a), unsuccessful search of the standard distribution (b)

Fig. 5 shows several search stages for the algorithm of the intensity distribution illustrated in Fig. 4a, in which there are three light traps for non-transparent microobjects.

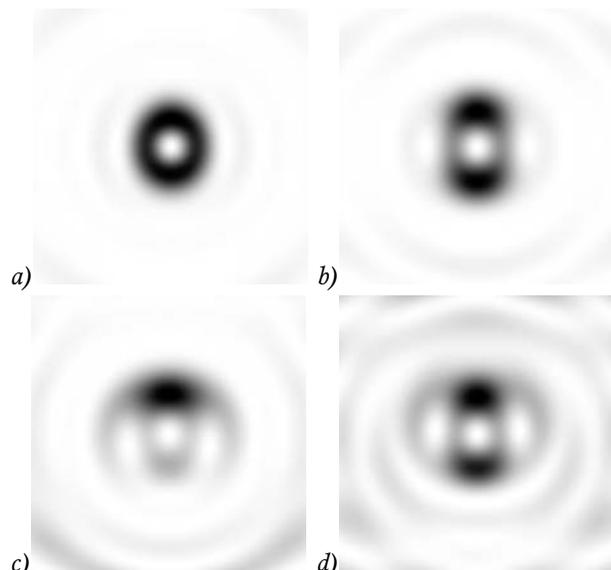
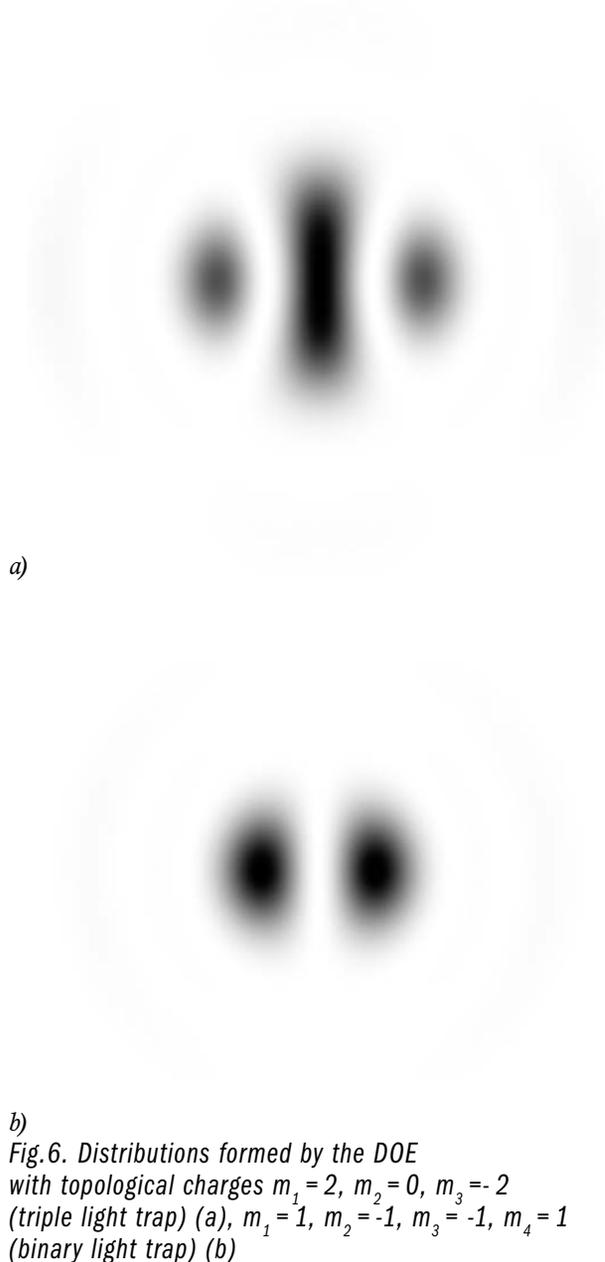


Fig. 5. Intermediate search phases

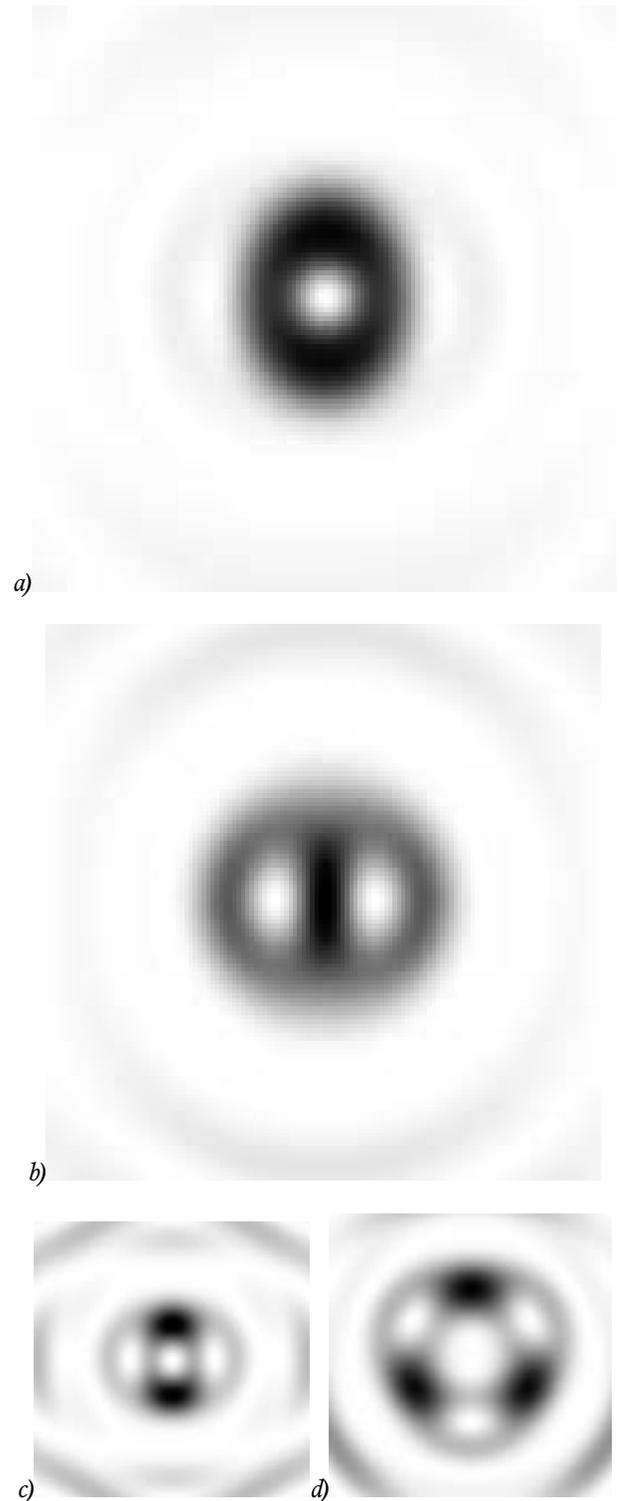
As can be seen from Fig. 5, as the result, the algorithm receives increasingly the same intensity distributions with every step, and finally comes to the standard intensity distribution. However, the search of the intensity distributions which form the light traps has a special feature: there is no need for the intensity distribution to exactly coincide with the standard. There is a set of areas in which the light traps are located (intensity minima or maxima), however there is no need to check all other distribution sites to conform to the standard. As the result, one more modification of the algorithm has been analyzed.

**3. The coordinatewise descent algorithm with minimizing the residual function based on the combination of the mean-square deviation and the first order moment of the intensity distribution by  $r$  in preset areas**

In order to improve the algorithm, we shall calculate the mean-square deviation only in those areas in which it is necessary to form the light trap. To avoid the algorithm enter a local minimum, the algorithm subsequently began iterations in several initial points succeeded by comparing the found solutions. Fig. 6 shows the search result for triple and binary traps for transparent microparticles, respectively.



Optimum values of topological charges for forming the preset traps have been found at 46 iterations. We can also obtain the light trap by specifying the coordinates of minima for transparent microobjects (Fig. 7) with the preset number and position of minima.



*b)*  
Fig.6. Distributions formed by the DOE with topological charges  $m_1 = 2$ ,  $m_2 = 0$ ,  $m_3 = -2$  (triple light trap) (a),  $m_1 = 1$ ,  $m_2 = -1$ ,  $m_3 = -1$ ,  $m_4 = 1$  (binary light trap) (b)

e)

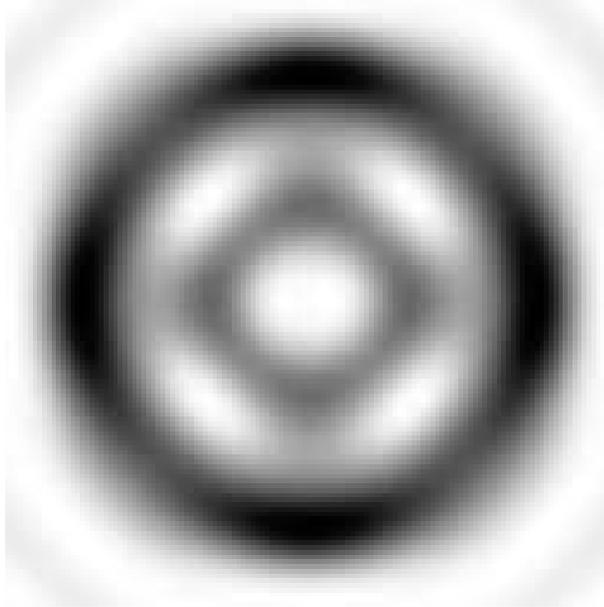


Fig. 7. Distribution formed by the DOE with topological charges  $m_1 = 3, m_2 = 1, m_3 = 1, m_4 = 1$  (single trap) (a), distribution formed by the DOE with topological charges  $m_1 = 2, m_2 = 0, m_3 = 2$  (binary trap) (b), distribution formed by the DOE with topological charges  $m_1 = 1, m_2 = 3, m_3 = 7, m_4 = 13$  (triple trap) (c), distribution formed by the DOE with topological charges  $m_1 = 5, m_2 = 5, m_3 = 2, m_4 = 2$ , the angle of zone rotation 180 (quadruple trap) (d), distribution formed by the DOE with topological charges  $m_1 = 6, m_2 = 2, m_3 = 6$  (quintuple trap) (e)

Thus, we can obtain almost any configuration of the traps by specifying coordinates of one or more intensity minima and receive a tool for capturing a group of microparticles.

## Conclusion

Based on the findings of the survey, we can draw a conclusion of the fundamental performance of the algorithm for calculating phase functions of the DOE which, due to forming of superpositions of the vortex beams, shall form the given configuration of the light traps. In this case the algorithm is based on specific features of the formation problem for the given set of the light traps, which lies in the fact that it requires the accurate determination of the light field just only in a small area around a definition point of the light trap. Hence the light field may be arbitrary throughout the whole rasterized image that gives the additional degree of freedom and enables to calculate the phase function of the DOE based on the coordinatewise descent algorithm in respect to the discrete coordinates (topological charges).

## Acknowledgements

The work has been performed with financial support from the Russian Science Foundation (grant 14-19-00114).

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