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THEORY OF COHERENT FOCUSERS

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Abstract—Two types of focusers are compared—one having the previously employed "integral" definition of intensity, and the other using a local definition and referred to as a coherent focuser. The solution of both problems reduces to integration of ordinary differential equations.

INTRODUCTION

A new type of optical device has recently been introduced to solve a series of scientific and technological problems, namely the focuser, designed to focus radiation into a line instead of a point. The task of focuser theory [1–4] is to determine the surfaces of lenses, mirrors, zone plates etc., which will transform a given primary wave U_0 into a field U focused into the required focal line $\vec{F}(\sigma)$, with given field intensity distribution $I(\sigma)$. On account of diffraction effects the width of the focusing zone (i.e. the diameter of the tube whose axis lies along the focal line) is obviously finite, and is determined by the ratio of the transverse focuser dimension to the wavelength.

We can conceive of the field intensity at the focal line in two ways: either as an integral quantity, namely as the energy flux per unit length across the cross-section of the focusing zone (the width of the focusing spot), or, as a local quantity represented by the squared field amplitude at the same focal line. This ambiguity in the definition of the intensity, together with differences in focuser design characteristics, as well as a series of other circumstances, have given rise to a number of variants in focuser theory.

At all events, if one ignores diffraction effects arising at the ends of the focal line, then the problem of determining the focuser surface, which is a function of two variables, reduces to finding a function of a single variable, to wit, the behaviour of the eikonal $s(\sigma)$ along the focal line (σ is the coordinate along the focal line). The functions $s(\sigma)$ and $\vec{F}(\sigma)$ are determined by the ray structure of the focuser field.

Below it will be demonstrated, through some examples, that in all variants of the theory the problem reduces to the solution of an ordinary differential equation for $s(\sigma)$. The differences in intensity definitions and focuser design will lead to changes in the differential equations for $s(\sigma)$.

RAY STRUCTURE OF FIELD FOCUSED ONTO FOCAL LINE

A circular cone of rays passes through each point of the focal line (Fig. 1).† One half of the cone corresponds to rays entering the focal line, the other half, to rays leaving it. The axis of the cone is tangent to the focal line, and the aperture angle is determined by the derivative $ds/d\sigma$ [5]:

$$\cos\omega(\sigma) = \frac{ds}{d\sigma}. \tag{1}$$

In the following calculations it is convenient to use a system of coordinates σ, ψ, ρ related to the ray structure, in which ψ is the angle characterizing the ray in the cone σ (namely, the angle reckoned from the osculating plane Λ to the focal line), ρ is the distance along the ray from the apex of the cone to the particular point M ($\rho \geqslant 0$ refers to rays entering, $\rho \leqslant 0$ to rays leaving the focal line).

The coordinate system σ, ψ, ρ is not radial, since the surface $\rho = \text{const.}$ is different from the surface $s = \text{const.}$ (*cf.* Eq. (5)).

The coordinates σ, ψ, ρ determine the position of the point M uniquely. The inverse statement

† There are cases in which not one but several (n) cones of rays pass through a point of the focal line. Such cases correspond to n-valued eikonal functions $s(\sigma)$. In the following calculation it is assumed that $s(\sigma)$ is a single-valued function of σ, so that only a single cone of rays passes through any point of the focal line.

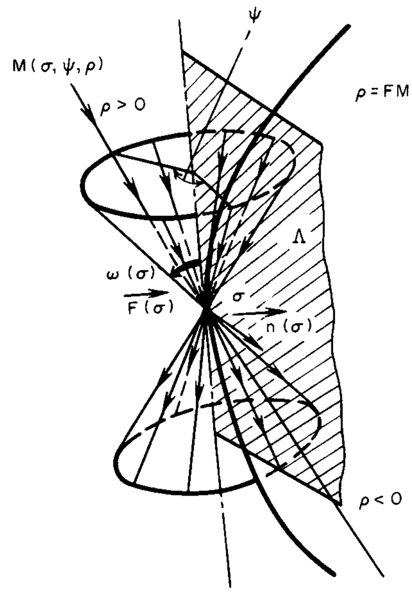

Fig. 1.

is not true in general. In a multiray region of space, where n rays pass through the point M, each point corresponds to n sets of coordinates σ, ψ, ρ. In ray-free regions the coordinates σ, ψ, ρ are complex.

The vector $\vec{M}(\sigma, \psi, \rho)$ is related to the position vector $\vec{F}(\sigma)$ on the focal line, and to the unit vector $\vec{q}_0(\sigma, \psi)$ of the ray on the cone, by

$$\vec{M}(\sigma, \psi, \rho) = \vec{F}(\sigma) + \vec{q}_0(\sigma, \psi)\cdot\rho. \tag{2}$$

In accordance with the previous definitions of σ and ψ we have

$$\vec{q}_0(\sigma, \psi) = \vec{\tau}(\sigma)\cdot\cos\omega(\sigma) + \vec{n}(\sigma)\cdot\sin\omega(\sigma)\cdot\cos\psi + \vec{b}(\sigma)\cdot\sin\omega(\sigma)\cdot\sin\psi, \tag{3}$$

where $\vec{\tau}$, $\vec{n}$, $\vec{b}$ are unit vectors of the tangent, normal and binormal to $\vec{F}(\sigma)$.

Since the cone axis orientation and the cone aperture angles are both functions of σ, the family of cones under given σ possesses a caustic-envelope surface for the radial ray field.† The equation of this surface is [5]:

$$\kappa(\sigma, \psi) = \left[\frac{s''(\sigma)}{\sin^2\omega(\sigma)} - \frac{\cos\psi}{\varepsilon(\sigma)\cdot\sin\omega(\sigma)}\right]^{-1}, \tag{4}$$

where κ is the distance along the ray (σ, ψ) from point $F(\sigma)$ of the focal line to the caustic surface, and $\varepsilon(\sigma)$ is the radius of curvature of the focal line.

The value of the eikonal of the ray field S at the point σ, ψ, ρ is

$$S(\sigma, \psi, \rho) = s(\sigma) - \rho(\sigma, \psi). \tag{5}$$

CONNECTION BETWEEN THE FOCUSER SURFACE FIELD AND THE INTENSITY $I(\sigma)$

The primary fields U_0 is transformed into the field $U(\sigma, \psi, \rho)$ on the focuser surface Σ and is focused onto the line $\vec{F}(\sigma)$. We assume we are given the focuser surface Σ, the distribution of the field U on it, as well as $\vec{F}(\sigma)$ and the value of the eikonal along it, $s(\sigma)$. Geometrical optics is valid on Σ. Hence U on Σ may be described via geometrical optics:

$$U = A(\sigma^*, \psi^*)e^{ikS_1(\sigma^*, \psi^*)},$$

where * denotes point coordinates on Σ, i.e. on Σ we take $\sigma = \sigma^*$, $\psi = \psi^*$. We also use the notation $\rho(\sigma^*, \psi^*) = r$.

† The second caustic surface is degenerate—it is the focal line itself.

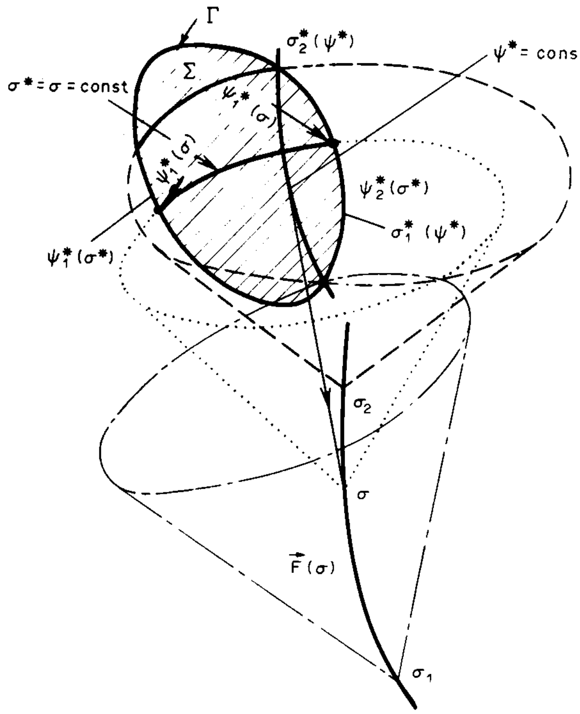

Fig. 2.

The contour Γ of the focuser surface is taken to be of the form (Fig. 2):

$$\sigma_1^*(\psi^*) \leqslant \sigma^* \leqslant \sigma_2^*(\psi^*); \qquad (\psi_{\min}^* \leqslant \psi^* \leqslant \psi_{\max}^*),$$

also

$$\psi_1^*(\sigma^*) \leqslant \psi \leqslant \psi_2^*(\sigma^*); \qquad (\sigma_{\min}^* \leqslant \sigma^* \leqslant \sigma_{\max}^*).$$

In the vicinity of the focal line, where geometrical optics does not apply, the field $U(\sigma, \psi, \rho)$ will be determined by using the Green–Kirchoff formula for the field $U(\sigma^*, \psi^*)$ on Σ. We assume the function $s(\sigma)$ in such as to ensure that only a single ray passes through any point of Σ, so that the caustic surface $\kappa(\sigma, \psi)$ does not intersect Σ, which is possible provided $\kappa < 0$ (the caustic surface is imaginary) or $\kappa > 0$ (Σ is reached before the caustic surface).

The field $U(\sigma)$, described by the Green–Kirchoff formula in terms of the integration variables σ^*, ψ^* of the observation points $\vec{F}(\sigma)$ on the focal line is given by

$$U(\sigma) = \frac{ik}{2\pi} \int_{\psi_{\min}^*}^{\psi_{\max}^*} d\psi^* \int_{\sigma_1^*(\psi^*)}^{\sigma_2^*(\psi^*)} A_1(\sigma^*, \psi^*) \cdot \frac{\cos\alpha(\sigma^*, \psi^*) + \cos\beta(\sigma, \sigma^*, \psi^*)}{2} \times \frac{e^{ik[S_1(\sigma^*, \psi^*) + d(\sigma, \sigma^*, \psi^*)]}}{d(\sigma, \sigma^*, \psi^*)} \times J(\sigma^*, \psi^*) d\sigma^*, \tag{6}$$

wherein α is the angle between the ray and the normal to Σ at (σ^*, ψ^*); β is the angle between the same normal and the vector joining point $\vec{F}(\sigma)$ to the integration point $r(\sigma^*, \psi^*)$; d is the distance between the observation and integration points; and J is the Jacobian for the coordinates (σ^*, ψ^*).

To integrate over σ^* we use the method of stationary phase. By Fermat's principle the stationary phase point is that point $\sigma^* = \sigma$, from which a geometrical optical ray arrives at $\vec{F}(\sigma)$ (*cf.* Fig. 2). Calculating the quantities α, β, d, J entering into (6) at $\sigma = \sigma^*$, neglecting the contribution of edge (contour) terms, and taking the finite σ^* integration limit to infinity, we obtain [6]:

$$U(\sigma) = \sqrt{\frac{k}{2\pi}} \int_{\psi_1^*(\sigma)}^{\psi_2^*(\sigma)} A_1(\sigma, \psi^*) \sqrt{\frac{|\kappa(\sigma, \psi^*) - r(\sigma, \psi^*)|}{|\kappa(\sigma, \psi^*)|}} \times e^{i[k(S_1(\sigma, \psi^*) + d(\sigma, \sigma^*, \psi^*)) - \operatorname{sgn}(s''(\sigma))]} \times \sqrt{r(\sigma, \psi^*)}\, d\psi^*.$$

On account of (2) and (3)

$$\kappa = \kappa(\psi^*, \sigma, s'(\sigma), s''(\sigma)),$$
$$r = r(\psi^*, \sigma, s'(\sigma)).$$

Since a cone of rays arrives at $F(\sigma)$, appropriate to a line of stationary phase $\sigma^* = \sigma$, then in accordance with (5) we have

$$s_1(\sigma, \psi^*) + d(\sigma, \sigma, \psi^*) = s(\sigma)$$

and, consequently,

$$U(\sigma) = \sqrt{\frac{ik}{2\pi}}\, e^{i[ks(\sigma) + \mathrm{sgn}(s(\sigma))]} \times \int_{\psi_1^*(\sigma)}^{\psi_2^*(\sigma)} A_1(\sigma, \psi^*) \sqrt{\frac{|r-\kappa|}{|\kappa|}} \cdot r\, d\psi^*. \tag{7}$$

Hence, the desired relation for the intensity $I_1(\sigma)$, taken as a local quantity in terms of square of the field amplitude at the focal line itself, will be expressed in terms of an integral over Σ along the line $\sigma^* = \sigma$:

$$I_1(\sigma) = |U|^2 = \frac{k}{2\pi}\left\{ \int_{\psi_1^*(\sigma)}^{\psi_2^*(\sigma)} A_1 \sqrt{\frac{|r-\kappa| \cdot r}{|\kappa|}}\, d\psi^* \right\}^2 \tag{8}$$

or, in symbolic form

$$I_1(\sigma) = H_1[\sigma, s'(\sigma), s''(\sigma), \psi_1^*(\sigma, s'(\sigma)), \psi_2^*(\sigma, s'(\sigma))]. \tag{8'}$$

Let us now calculate the intensity $I_2(\sigma)$, using its alternative, integral definition [1] in terms of the energy flux across an area whose width is the focal spot diameter, and whose height is $d\sigma$. By conservation of energy this flux is equal to the energy current across a strip lying between σ and $\sigma + d\sigma$ on Σ. This definition gives us

$$I_2(\sigma) = \int_{\psi_1^*(\sigma)}^{\psi_2^*(\sigma)} A_1^2(\sigma, \psi^*) \cdot J(\sigma, \psi^*) \cdot \cos\alpha(\sigma, \psi^*)\, d\psi^*$$
$$= \sin\omega(\sigma) \cdot \int_{\psi_1^*(\sigma)}^{\psi_2^*(\sigma)} A_1^2 \frac{|\kappa - r|}{|\kappa|} r \cdot d\psi^*, \tag{9}$$

so that

$$I_2(\sigma) = H_2[\sigma, s'(\sigma), s''(\sigma), \psi_1^*(\sigma), \psi_2^*(\sigma)]. \tag{9'}$$

THE EQUATION FOR $S_1(\sigma^*, \psi^*)$

In the previous section we considered the direct problem, so that $s(\sigma)$ was assumed known on the focal line. This means the grid coordinates σ^*, ψ^* on Σ, as well as the functions $S_1(\sigma^*, \psi^*)$, $A_1(\sigma^*, \psi^*)$ giving the field $U_1 = A_1 e^{ikS_1}$ passing the focuser, were all taken as known, and what we were after was the intensity, whether $I_1(\sigma)$ or $I_2(\sigma)$.

In the following discourse we treat the inverse problem: given $I(\sigma)$, we would like to determine the function $S_1(\sigma^*, \psi^*)$. From the above discussion it follows that it suffices to calculate the function $s(\sigma)$. We assume that the function A_1 on Σ is known. This formulation of the inverse problem corresponds to a focuser in the shape of a thin (possibly a zoned) lens, which may be construed as a phase transformer converting the initial wave $U_0 = A_0 e^{ikS_0}$ falling on Σ into the wave $U_1 = A_0 e^{ikS_1}$, without amplitude change. The change in the eikonal during the phase transformation,

$$\varphi(\sigma^*, \psi^*) = S_1(\sigma^*, \psi^*) - S_0(\sigma^*, \psi^*)$$

determines the focuser structure, i.e. the coordinate dependence of its optical thickness.

We must solve either Eq. (8′) or (9′) (depending on the definition of the intensity $I(\sigma)$) for the function $s(\sigma)$. These are ordinary differential equations (not solved with respect to the leading term $s''(\sigma)$). Since only s' and s'', and not the function s itself, enter into each of (8′) and (9′), these are

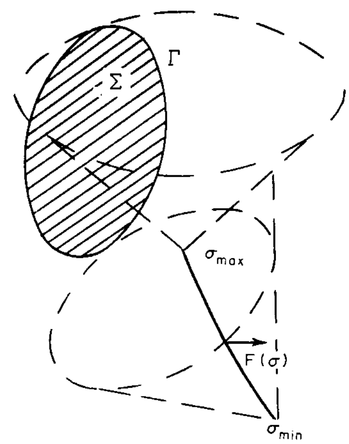

Fig. 3.

first-order equations for the function $\eta(\sigma) \equiv s'(\sigma)$, of the type:

$$c_1 \cdot I_1(\sigma) = H_1(\sigma, \eta, \eta'); \tag{10}$$

$$c_2 \cdot I_2(\sigma) = H_2(\sigma, \eta, \eta'), \tag{10'}$$

in which the intensities I_1 or I_2 are normalized by the numbers c_1, c_2. These are determined, as are the initial values η_{min}, by the boundary conditions: the cone of rays leaving the two ends σ_{min} and σ_{max} of the focal line must be tangential (not intersecting) to the contour Γ of the focuser surface Σ (Fig. 3).

The solution of (10) or (10′) subject to the specified boundary conditions determines the eikonal distribution $s(\sigma)$ along the focal line,† which in turn gives the ray structure of the field U. This allows one to write the equation for Σ in the form $r = r(\sigma, \psi)$. By means of Eq. (5) one then calculates $S_1(\sigma^*, \psi^*)$, which suffices to determine the thickness of a thin zoned focuser. It is important to note that the physically realized solutions are only those for which each point of the Σ surface corresponds to a single ray. If the solution does not fulfil this requirement the problem must either be reformulated, or one must alter the mutual disposition of the focal line and the focuser, or finally, make an additional incision on Σ, which is equivalent to modifying the contour Γ [7].

In general, Eqs (10) and (10′) are solved numerically. In numerical integration algorithms one must calculate at each stage the numerical value of the derivative η', for which (10) and (10′) serve as transcendental equations.

The differences between (10) and (10′), stemming from different handling of the intensity, are manifested in different solutions for $s'(\sigma)$ and, consequently, $S_1(\sigma^*, \psi^*)$. In other words, different intensity definitions result in different focuser structures. We illustrate these differences in the simplest example of a focuser whose geometry is shown in Fig. 4. The focuser is plane, its contour Γ is an isosceles trapezium with bases 1 and 0.1 and height 1. The focal line is a straight line which is parallel to the plane of the focuser and symmetric with respect to its lateral sides. The distance f from the focal line to the focuser is 10 or 100. Its length is 0.5 ($\sigma_1 = -0.25$; $\sigma_2 = 0.25$). The amplitude distribution over the focuser plane, $A(y) = [\alpha y + \beta]^{-1/2}$ ($\alpha = 0.45$; $\beta = 0.275$) corresponds (*cf.* below) to a linear law of reflection for the "integral" definition of the intensity. Since in this example $f \gg 1$, the paraxial approximation is valid, so that one can put $r \simeq f$; $\kappa \simeq (s'')^{-1}$ and write (9) in the form:

$$s'' \cdot f = \frac{c_2}{f} - 1. \tag{11}$$

The two constants entering into the solution of (11) (c_2 and a constant of integration) are fixed by the boundary conditions: the cones corresponding to the ends of the focal line emanate from below and above the trapezium bases.

† The solution for $s(\sigma)$ must be a single valued function σ, so that only one cone of rays passes through each point σ.

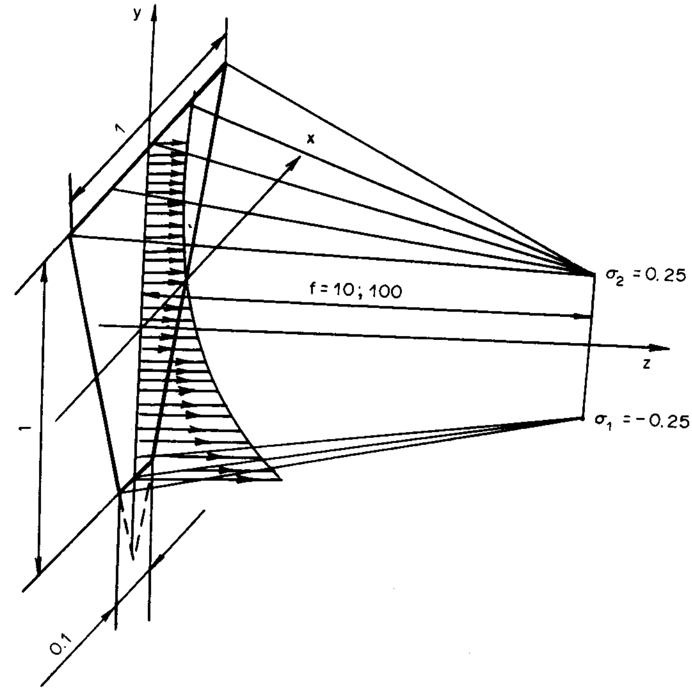

Fig. 4.

The solution of (11) is the linear function:

$$s'(\sigma) = A_2(\sigma - \sigma_1) + B_2, \tag{12}$$

where

$$A_2 = \frac{1}{\sigma_2 - \sigma_1}\left[\frac{y_2 - \sigma_2}{\sqrt{(y_2 - \sigma_2)^2 + f^2}} - \frac{y_1 - \sigma_1}{\sqrt{(y_1 - \sigma_1)^2 + f^2}}\right]; \qquad B_2 = \frac{y_2 - \sigma_2}{\sqrt{(y_2 - \sigma_2)^2 + f^2}}.$$

Equation (8), corresponding to the local definition of the intensity,

$$(1 + f\cdot s'')\cdot(\sigma + f\cdot s') = \frac{2\pi c_1}{kf\alpha}, \tag{13}$$

gives a solution

$$s' = [A_1\cdot(\sigma - \sigma_1) + B_1]^{1/2} - \sigma/f, \tag{14}$$

which differs from (12).

We note that because of the boundary conditions, the solutions (12) and (14) (Fig. 5) coincide at the points $\sigma = \pm 0.25$ designating the ends of the focal line. The largest difference between (12) and (14) is observed in the middle of the focal line, and is more pronounced for the short focus system ($f = 10$).

We remark that in the paraxial approximation $s'(\sigma)$ is independent of the shape of the surface Σ and depends only on the form of the contour Γ (more precisely on its projection on the plane of the primary wavefront). But the phase structure function of the focuser, (σ^*, ψ^*), does depend on the form of the surface as well as on the function $s'(\sigma)$.

SURFACE DESIGN OF REFLECTING FOCUSER

We assumed above that the focuser was thin (e.g. zoned) and that its surface Σ was given. These assumptions led to a differential equation for $s(\sigma)$ containing only the derivatives $s'(\sigma)$ and $s''(\sigma)$. If the focuser is thick then, in addition to these s will enter the equation.

We demonstrate this in the problem of synthesizing a reflecting focuser (Fig. 6). The function to be calculated is then $R(\sigma, \psi)$, the mirror surface, which transforms the primary spherical wave

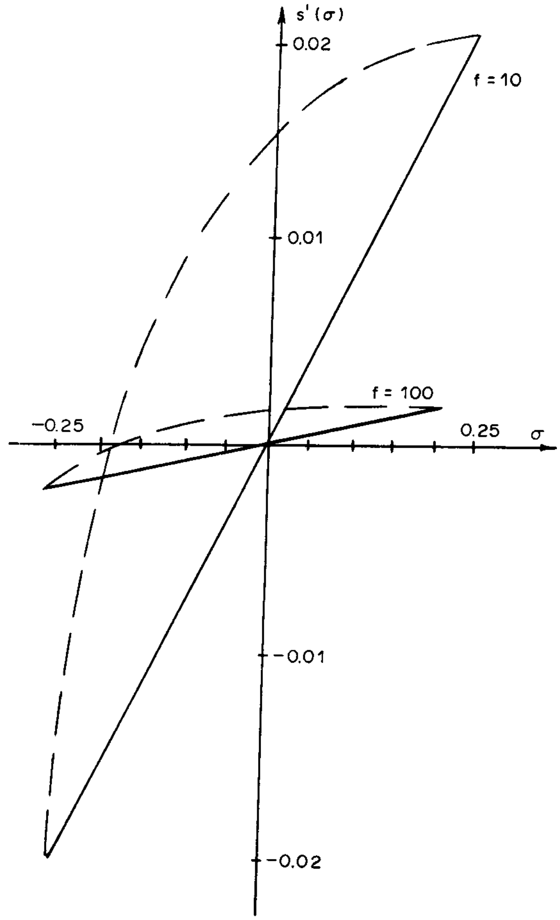

Fig. 5. ——— For integral definition of the intensity. ----- For local definition of the intensity.

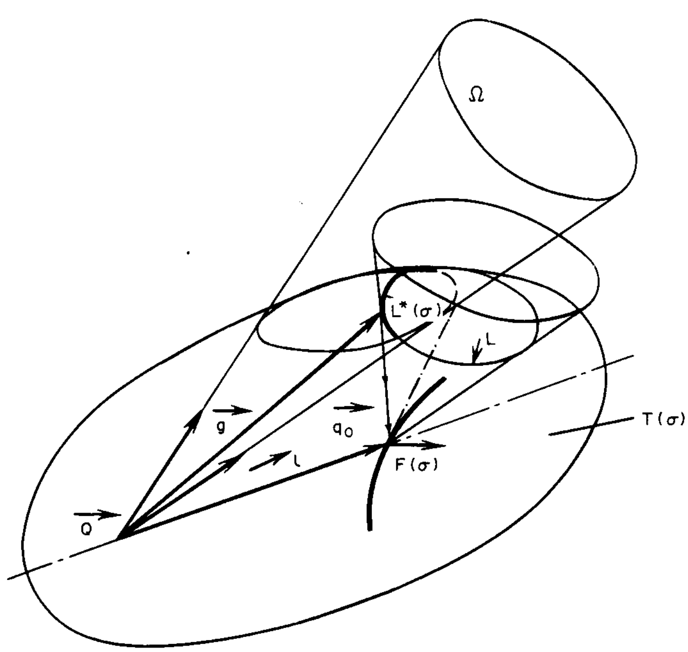

Fig. 6.

(whose centre in Fig. 6 is the point $\vec{Q}$) into the field U focused onto the focal line $\vec{F}(\sigma)$, along which we are given the intensity $I(\sigma)$. The initial field U_0 is directed. It is nonvanishing in the solid angle whose boundary is the cone Ω (equivalent to the contour Γ in the previous problem), wherein it is characterized by the diagram $\Phi(\Theta, \varphi)$. As before, we first solve the direct problem, i.e. we look for $I(\sigma)$, assuming $s(\sigma)$ is given.

We consider a line (or strip) $\mathscr{L}$ on the mirror transforming a ray of the initial spherical wave into a cone of rays arriving at a point on the focal line. The line $\mathscr{L}$ is formed by the intersection of this cone with the ellipsoid of rotation $T(\sigma)$ (Fig. 6), whose foci are at points $\vec{Q}$ and $F(\sigma)$. The length of the optical path, i.e. the major axis of the ellipsoid, is equal to the sum of $s(\sigma)$ and an arbitrary constânt h [8].

The position of one of the foci, namely $\vec{F}(\sigma)$, and the optical path $s(\sigma)+h$, are therefore both functions of σ. The family of lines $\mathscr{L}$ generated by the parameter σ forms the mirror surface.

Obviously one should use for the mirror only the part $\mathscr{L}^*$ of the lines lying within the cone Ω, where the amplitude of the initial wave is nonzero.

The parametric representation of the mirror surfaces in terms of the coordinates σ, ψ has the form:

$$R(\sigma, \psi, s(\sigma), s'(\sigma)) = \frac{p(\sigma)}{1 - e(\sigma)\cdot(\vec{l}_0(\sigma)\cdot\vec{q}_0(\sigma, \psi))}, \tag{15}$$

where

$R(\sigma, \psi)$ is the distance along the ray ψ from the focal line point $\vec{F}(\sigma)$ to the mirror;

$p(\sigma) = [(s(\sigma)+h)^2 - l(\sigma)^2]/2[s(\sigma)+h]$ is the focal parameter of the ellipsoid;

$e(\sigma) = |l(\sigma)|/[s(\sigma)+h]$ is the eccentricity;

$|\vec{l}(\sigma)| = |\vec{F}(\sigma) - \vec{Q}|$ is the distance between the ellipsoid foci;

$\vec{q}_0(\sigma, \psi)$ is the unit vector of the reflected ray in the cone (3);

$\vec{l}_0(\sigma) = \vec{l}(\sigma)/|\vec{l}(\sigma)|$ is the unit vector along the ellipsoid axes.

After a calculation which is similar to that performed above, we may write the intensity $I(\sigma)$ on the focal line by means of the Green–Kirchoff formula:

$$I(\sigma) = \frac{k}{2\pi}\left\{\int_{\psi_1^*(\sigma)}^{\psi_2^*(\sigma)} \frac{\Phi[\Theta(\sigma, \psi^*), \varphi(\sigma, \psi^*)]}{|s(h)+h-R(\sigma, \psi^*)|}\sqrt{R(\sigma, \psi^*)\frac{|\kappa(\sigma, \psi^*) - R(\sigma, \psi^*)|}{\kappa(\sigma, \psi^*)}}\,d\psi^*\right\}^2. \tag{16}$$

The angular coordinates Θ and φ, which determine the directivity diagram, are expressed in terms of the coordinates σ, ψ via the law of specular reflection (*cf.* Fig. 6):

$$\vec{g}(\Theta, \varphi) = \vec{l}(\sigma) - \vec{q}(\sigma, \psi)\cdot R(\sigma, \psi), \tag{17}$$

where $\vec{g}(\Theta, \varphi)$ are vectors connecting points on the source to points on the mirror. The solution of the direct problem is thus given by Eqs (15) and (16). (Equations (15) and (16) contain a free parameter h, so that the solution is not single valued.)

Let us now turn to the inverse problem. This time $I(\sigma)$ is given and we use Eq. (16) to solve for $s(\sigma)$. Equation (16) becomes a second-order implicit differential equation for $s(\sigma)$:

$$c_3\cdot I(\sigma) = H_3(\sigma, s, s', s''). \tag{18}$$

Unlike (8′) and (9′) here the eikonal itself appears, in addition to the first- and second-order derivatives, so that the second-order differential equation is not solved with respect to the leading derivative. The sought after solution $s(\sigma)$ must satisfy the boundary condition that the mirror line corresponding to the ends of the focal line $\sigma = \sigma_1$ and $\sigma = \sigma_2$ must be tangential to the boundary cone Ω of the initial field without intersecting it. This condition determines the values of $s(\sigma_1)$, $s'(\sigma_1)$ and $s(\sigma_2)$, $s'(\sigma_2)$. For the above choice of initial conditions on the focuser surface $s(\sigma)$ one obtains lies within the cone Ω, and its limiting contour Γ lies on the cone itself.

We obtain a second-order differential equation in $s(\sigma)$ also when the integral definition of the intensity is used. The initial conditions and a constant c_3 are fixed by equating the energy flux in the cone Ω of the initial wave to that across the focal line.

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