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BINARY ELEMENTS OF BESSEL OPTICS

A. E. BEREZNII and I. N. SISAKYAN

Abstract—Experimental results are presented for high-order Bessel transformations applied to wave fields with circular symmetry. The optical transformations are based on binary phase holograms.

Present-day optics already employs a whole series of classical integral transformations helpful in solving many important problems. Pride of place goes to Fourier optics, but Fourier-related transformations such as the Hilbert and Mellin transforms should also be mentioned [1, 2]. The appearance of new computer-aided designs of optical elements [3] has enhanced enormously the class of integral transforms and other operations on light fields realizable by optical methods.

One of the promising transformations is the Bessel transform (a generalized Hankel transform [4]). On the whole the possibilities offered by Bessel optics resemble those of Fourier optics, but the main area of application is to optical fields (beams) with circular symmetry. The axial symmetry of Bessel-optics problems is similar to the axial symmetry characteristic of many optical problems, and determines its area of application. In the present work we demonstrate the possibility of realizing all such operations with symmetric beams, analogous to the operations of Fourier optics for beams without circular symmetry.

Such fields are essentially one dimensional. Because of this, one-dimensional techniques applied to them (integral transformations and series expansions in terms of single variable functions) are much more effective than two-dimensional methods.

Bessel optics is realized by expanding plane optical fields with circular symmetry in a series of Bessel functions of fixed order and various arguments [5, 6], similar to Fourier series.

The Bessel transformations is an inverse integral transform and is the same as its inverse, so that the Bessel harmonics become δ-functions. This has the consequence that spatial filtering can be carried out just as in Fourier optics [1]. In order to carry out a Bessel transformation on the optical field at the entrance plane, it suffices to have a lens performing a Fourier transformation, and an $e^{in\Theta}$-type phase filter in the form of a binary hologram which encodes the requisite phase function and is positioned at the entrance plane of the system. Such filters are produced by methods explained in [7], designed on a computer and developed on a photomaterial by means of a "P-1700" type scanning device. Estimates have been given of the effect of errors in phase discretization and quantization (incurred during manufacture) on element operation.

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