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OPTICAL PHASE ELEMENTS FOR ANALYTIC TRANSFORMATION OF COORDINATES

A. N. ANDREEV, A.,E. BEREZNII and I. N. SISAKYAN

Abstract—The paper presents experimental data on optical implementation of a two-dimensional analytic coordinate transformation in pictures by means of two-dimensional phase holograms. The correctness of the chosen mathematical model is established. The required degree of circuit adjustment and transformation accuracy parameters are estimated. Transformation of cartesian coordinates into polar–logarithmic ones (polar angle, logarithm of the polar radius) was carried out experimentally.

In optical information processing with coherent and incoherent light a typical operation that occurs is an image coordinate transformation, to wit, a transformation of the light field within which points (x, y) in the entrance plane are mapped to the pair (u, v) in the exit plane of the system, where $u = u(x, y)$, $v = v(x, y)$, and the correspondence is construed in the sense of geometrical optics. What this means for a Fresnel zone is that (x, y) is the point of entry of the beam, while (u, v) is the point of exit. In the Fraunhofer region the correspondence is established by the method of stationary phase. The problem of deriving the desired transformation in a coherent field by means of synthesized holograms has been repeatedly discussed in the literature (*cf.* [1]). The construction of the desired synthetic phase hologram boils down to solving a system of first-order partial differential equations. The question of the integrability of these equations, namely, whether the required phase hologram may be obtained in principle, has been clarified in [2].

In the present note we examine experimentally the transformation of cartesian to polar–logarithmic coordinates, important for a series of applications, in particular for holographic pattern recognition [3] (by means of two-dimensional correlations) that are invariant to rotation, changes of scale and shifts in the incoming signal (and not just to shifts which normally occur in holographic correlators). Solving this problem in various ways is one of the most popular current topics in optical information processing.

A transformation scheme for picture coordinates by utilizing the frequency surface of a lens, was presented in [2]. If the phase function of the optical element is denoted by $f(x, y)$, we have in the paraxial region,

$$u = x + lf_x(x, y), \qquad v = y + lf_y(x, y),$$

where x, y are beam cartesian coordinates in the forward focal plane of the lens, and l is the distance between the (x, y) and (u, v) planes.

Although the scheme chosen is the simplest one experimentally, it is not optimal as far as sensitivity to realignment and hologram production precision are concerned, since the ray matrix of such a system rearranges the position of the coordinates and the directions in the phase space of the system. Such a transformation may also be implemented in a different optical system containing additional lenses, which does not suffer from this fault.

Holograms were produced with the following phase function

$$f(x, y) = C(xt + y \ln(r/r_0)),$$

where t is the polar angle in the (x, y) plane, i.e. $x = r \cos t$, $y = r \sin t$. This defines the transformation in question with different parameters C, r_0 and various hologram encoding procedures.

The holograms were produced by means of a p-1700 scanning device. Since the spatial-band production of this system is not large (about a 1000 was used, where the limit was 10000 elements per stroke), working near and even beyond the Nyquist limit was tried (sometimes this worked, thanks to the nature of the transformation). The best results were obtained by a cosine encoding, such that the carrier frequency equalled the Nyquist limit.

The experimental results proved that optical systems of the required quality could be constructed from modern elements, capable of carrying out arbitrary analytic coordinate transformations.

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