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USE OF A NONLINEAR THERMAL REFLECTION EFFECT TO CONTROL SPATIAL IRRADIANCE DISTRIBUTION

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Abstract—The reflection factor at the interface between two media can be controlled by heating with an auxiliary illuminating beam. Analytical expressions are derived for the temperature of the interface under instantaneous and stationary heating. For the case of stationary heating, the inverse transform expression is obtained. It indicates a practical possibility for realizing virtually any spatial distribution of the reflection factor.

The effect of nonlinear thermal reflection (NTR) is essentially a variation of reflectance due to a thermal nonlinearity induced by radiation striking the interface between two media. It has been studied in depth both theoretically and experimentally [1–5]. Under NTR, reflection readily fits a hysteresis mode, and interferometers with thermal nonlinearity exhibit a bistable mode. Applications reported for NTR include Q-switching [1, 6], optical shutters [3] and optical flip-flops [5].

Chernov and Shepelev [1] and Zuev *et al.* [7] suggested that the effect of NTR should be used to control the spatial distribution produced by irradiance equipment. Such devices are much needed in adaptive optics and optoelectronics.

In this paper we present exact solutions for two more important situations of harnessing NTR to control the spatial distribution of irradiance. We perform also a numerical analysis of reflecting type transparencies.

If the temperature of an isotropic medium changes by ΔT, and the density by $\Delta\rho$, the refractive index varies by

$$\Delta n = \left(\frac{\partial n}{\partial T}\right)_\rho \Delta T + \left(\frac{\partial n}{\partial \rho}\right)_T \Delta\rho. \tag{1}$$

The temperature of the media that form the interface is described by a system of heat transfer equations, which in the frequent case of radial symmetry has the form

$$\begin{aligned} &\frac{1}{a_2}\frac{\partial T_2(r,z,t)}{\partial t} - \nabla^2 T_2(r,z,t) = \frac{\sigma}{c} I(r,t) R(T,\rho,t) \mathrm{e}^{-\sigma z}, \\ &\frac{1}{a_1}\frac{\partial T_1(r,z,t)}{t} - \nabla^2 T_1(r,z,t) = 0. \end{aligned} \tag{2}$$

The associated boundary conditions of the fourth kind are

$$\begin{aligned} &T_1(r,0,t) = T_2(r,0,t), \\ &\frac{\partial T_1(r,0,t)}{z} = \frac{\lambda_2}{\lambda_1}\frac{\partial T_2(r,0,t)}{z}, \end{aligned} \tag{3}$$

where $a_{1,2}$ is the thermal diffusivity, c is the heat capacity and $\lambda_{1,2}$ is the thermal conductivity.

The right-hand side of the first equation in (2) describes the heating of medium 2 by a radiant flux $I(r, t)$. The distribution of the irradiance over the z axis is given by the Bouguer exponential $\mathrm{e}^{-\sigma z}$, and the factor $R(T, \rho, t)$ takes into account the reflection factor.

Equations (1) and (2) taken together with the boundary conditions (3), initial conditions and the state equation form a closed system that is sufficient to define the temperature and the reflection factor. This rather complex problem has not been solved in its general form. Important particular solutions corresponding to the one-dimensional problem in the limiting cases of short and long time have been investigated by Chernov and Shepelev [1] and Boiko *et al.* [2].

The following investigation is focused on the three-dimensional problem with radial symmetry

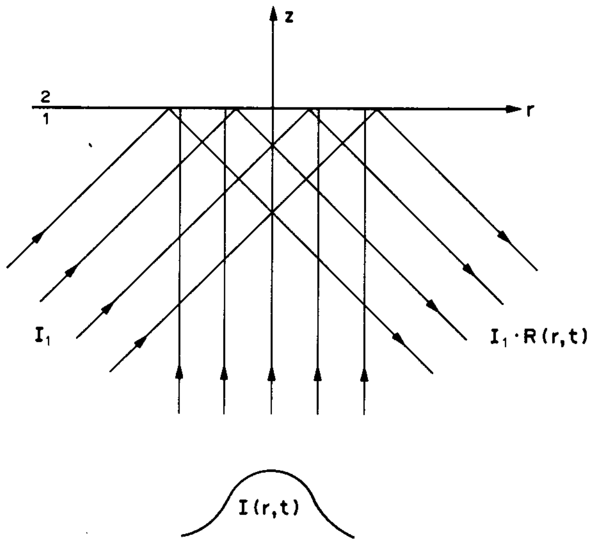

Fig. 1. Control of the reflection factor for beam I_1 by radiation $I(r, t)$ reflected from the interface between absorptionless medium 1 and absorbing medium 2.

using the approximation of an instantaneous propagation of sound, i.e. for times not higher than l/v, where l is the minimum dimension of the heated region and v is the velocity of sound. Under the circumstances, the refractive index is controlled solely by temperature and Eq. (1) reduces to

$$\Delta n = \frac{\mathrm{d}n}{\mathrm{d}T}\,\Delta T, \tag{4}$$

i.e. at each point the refractive index is a function of temperature. Where the temperature varies slowly over the wavelength, which corresponds to the actual situation, the reflection factor is described by the ordinary Fresnel formulae.

Consider the case of external heating (Fig. 1) in which the variation of the temperature relief and consequently the reflection factor for beam I_1 is controlled by the radiant flux $I(r)$. The reflectance coefficient for the heating beam $I(r)$ is practically independent of temperature because of almost normal incidence of this irradiance on the interface [1].

In the stated approximation, $R \equiv 1$.

(i) $I(r, t) = E(r)\delta(t)$, *instantaneous heating*

The solution of the system of equations (2) subject to (3) was obtained by sequential application of the Laplace and Hankel transformations, *viz.*,

$$T_{1,2}(r, z, t) \underset{L}{\rightarrow} T_{1,2L}(r, z, s) \underset{H}{\rightarrow} T_{1,2H}(\xi, z, s) \equiv Y_{1,2}.$$

The transforms carry the system of partial differential equations for $T(r, z, t)$ to the system of linear differential equations in $Y_{1,2}$

$$sY_1 - \mathrm{e}^{-\sigma z}H = a_1(-\xi^2 Y_1 + \mathrm{d}^2 Y_1/\mathrm{d}z^2),$$
$$sY_2 = a_1(-\xi^2 Y_2 + \mathrm{d}^2 Y_2/\mathrm{d}z^2),$$

related by the conditions

$$Y_1(0) = Y_2(0),$$
$$Y_1'(0) = \frac{\lambda_2}{\lambda_1}\,Y_2'(0).$$

Solving this system and performing the inverse transformations of Hankel and Laplace at $z = 0$,

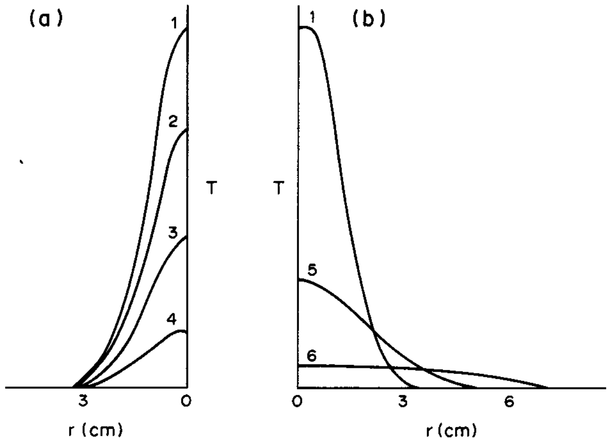

Fig. 2. Temperature distributions at $a = 4 \times 10^{-3}$ cm^2 s^{-1} for different values of at and σ. (a) $\sigma = 30$ cm^{-1}: (1) $at = 0$, (2) $at = 10^{-4}$ cm^2, (3) $at = 10^{-3}$ cm^2, (4) $at = 10^{-2}$ cm^2. (b) $\sigma = 0.1$ cm^{-1}: (1) $at = 0$, (5) $at = 1$ cm^2, (6) $at = 10$ cm^2.

we obtain the expression for temperature at the interface

$$T(r,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}s \int_0^{\infty} \mathrm{d}\, \frac{H(1 - \sigma/\xi^2 + s/a_1)J_0(\xi r)\exp(st)}{a_1\left(\xi^2 + \dfrac{s}{a_1} - \sigma^2\right)\left(1 + \dfrac{\lambda_2}{\lambda_1}\sqrt{\dfrac{\xi^2 + s/a_1}{\xi^2 + s/a_2}}\right)}, \tag{5}$$

where

$$H(\xi) = \int_0^{\infty} E(r)J_0(\xi r)\,\mathrm{d}r,$$

and J_0 is the Bessel function.

At $a_{1,2} = a$, for a Gaussian distribution of intensity

$$I(r,t) = \frac{E_0}{2\pi\rho}\exp(-r^2/2\rho^2)\delta(t)$$

the solution takes the form

$$T(r,t) = (\rho^2 + at)^{-1}\,\frac{E_0\sigma\exp(-\sigma^2 at)}{1 + \lambda_2/\lambda_1}\,[1 - \phi(\sigma\sqrt{at})]\exp -\left[\frac{r^2}{2(\rho^2 + at)}\right].$$

The reflection factor is determined from the above expression for temperature, Eq. (4) and Fresnel's formulae. Figure 2 shows the radial temperature distributions at various instants in time for Bouguer's linear absorption coefficients of 30 cm^{-1} at (a) and 0.1 cm^{-1} at (b). For $\sigma = 30$ cm^{-1} the dynamic behaviour of the reflection factor is controlled mainly by the transport of heat normal to the interface, and the profile of reflection factor remains practically the same. Thus at $a = 4 \times 10^{-3}$ cm^2 s^{-1}, which corresponds to the actual values, a sizeable change in reflection factor occurs in 10 ms for $\sigma = 30$ cm^{-1} and in 10 s for $\sigma = 0.1$ cm^{-1}.

(ii) *$I(r,t) = I(r)$, stationary heating*

In the stationary case, the time derivatives in (2) vanish and the initial conditions are irrelevant. The expression for the stationary distribution of temperature derived similar to (5) has the form

$$T(r) = \frac{\sigma}{a_2 c(1 + \lambda_2/\lambda_1)}\int_0^{\infty} \frac{rH(\xi)}{\sigma + \xi} J_0(\xi r)\,\mathrm{d}\xi,$$

whence

$$I(r) = \frac{ca_2}{\sigma}(1 + \lambda_2/\lambda_1)\int_0^{\infty} \mathrm{d}\xi\,\xi^2(\xi + \sigma)J_0(\xi r)\int_0^{\infty} J_0(\xi r')T(r')r'\,\mathrm{d}r'. \tag{6}$$

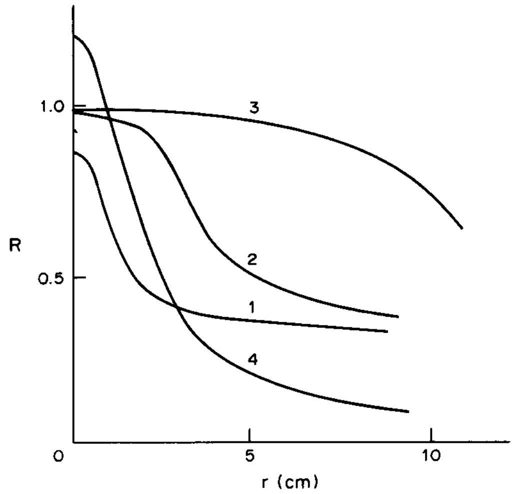

Fig. 3. Distributions of reflection factor and temperature in stationary heating for a Gaussian distribution of beam $I(r)$ ($\rho = 1$ cm and $\sigma = 30\ \text{cm}^{-1}$) and different powers of the heating beam. (1) 1 W, (2) 3 W, (3) 10 W, (4) steady-state temperature profile.

This expression is the solution of the inverse problem concerned with the evaluation of an intensity distribution which induces a given distribution of temperature $T(r)$. The integral (6) exists for a wide class of functions $T(r)$. Because the temperature uniquely defines the reflection factor the existence of (6) implies that one may furnish the necessary distribution of heating intensity to realize a given transparent element.

The settling time to a steady-state distribution of temperature is defined by the temperature diffusivity a_2 and the geometry of the problem. For high absorption, this time is in the order of r_0^2/a_2, where r_0 is the typical radius of the "soft" aperture.

The stationary heating behaviour is illustrated in Fig. 3 depicting the stationary distribution of reflection factor needed for different radiant powers and the profile of the steady-state temperature distribution. The *s*-polarized heating radiation has been assumed to have a Gaussian profile, the angle of incidence being 67°, i.e. 0.8° short of the angle for total internal reflection.

In summary, the effect of nonlinear thermal reflection allows one to realize a transparent element with any given transmittance profile that can be varied in time. The evidence obtained in this study may be readily extended to the case of an interferometer with thermal nonlinearity.

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