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WAVEGUIDES

DYNAMICS OF NONSOLITON PULSES IN A NONLINEAR TRANSMISSION CHANNEL

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Abstract—The paper reports a study of the evolution of ultrashort nonsoliton pulses travelling in a nonlinear single-mode waveguide. The intensity maximum and halfwidth of nonsoliton signal envelopes exhibit oscillatory and monotonous behaviour.

An application of ultrashort pulses is associated with the transmission of large amounts of information via optical fibres. Due to the short length ($\leqslant 1$ mm) of these pulses, they are susceptible to dispersion spreading and deterioration of the pulse repetition rate when propagated through a nonlinear lightguide. However, the stabilization of such pulses may be much improved by using nonlinear effects in opposition to the dispersion ones.

Practical transmission through optical fibres requires a special system of signals that play the role of elementary symbols. The early designs of such communication lines have been associated with solitons [1] whose envelopes do not contain free parameters. However, at a distance of several dispersion lengths, typical of integrated computer systems, where nonsoliton signals are stable, transmission with nonsoliton signals allows the free parameters to be used for message encoding. This possibility may be of interest for transmitting signals of multicharacter logic and for using more intricate codes than the traditional binary code.

When a strong short pulse propagates in a single-mode optical fibre whose refractive index is the square function of the field amplitude

$$n = n_0 + n_2|\mathbf{E}|^2$$

and there is negligibly small attenuation in the region of anomalous dispersion, the evolution of the normalized complex envelope of the pulse $\psi(\tau, \eta) = E(\eta, \tau)/E(0, \tau)$ is described by the nonlinear Schrödinger equation

$$\mathrm{i}\frac{\partial\psi}{\partial\eta} + \frac{\partial^2\psi}{\partial\tau^2} + \kappa|\psi|^2\psi = 0$$

with the initial condition $\psi(0, \tau) = \psi_0(\tau)$ defined by the pulse at the input to the fibre. Here, η and τ are the normalized coordinates related to the longitudinal spatial coordinate z and real time t as

$$\eta = \frac{z}{L}, \qquad \tau = \frac{t - z/v_0}{T_0},$$

where v_0 is the group velocity, L_ω and T_0 are the characteristic dimensions, and

$$\kappa = L_\omega \omega c^{-1} n_2 |\mathbf{E}|_m^2 \tag{1}$$

is the nonlinearity parameter defined by the properties of fibre material and by the launched pulse.

In Eq. (1), we neglect dispersion terms higher than the second order, the dependence of the group velocity on intensity, and attenuation.

The well-known stationary solution of Eq. (1) in the soliton of the envelope is

$$|\psi(\eta, \tau)|^2 = \left(\cosh^{-1}\sqrt{\frac{\kappa}{2}}\cdot\tau\right). \tag{2}$$

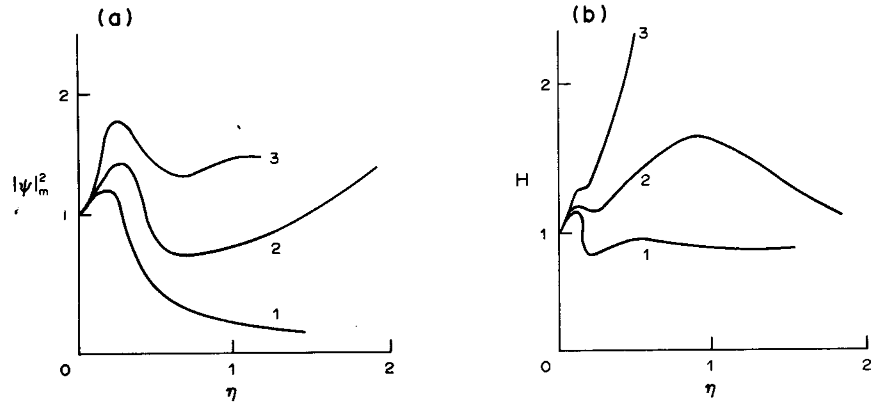

Fig. 1. Evolution of the parabolic pulse (3) at $\nu = 2$. Curves 1, 2 and 3 correspond to $\kappa = 0$, $\kappa = 2.5\kappa_{cr}$, and $\kappa = 4\kappa_{cr}$, respectively. The width at half-maximum H is measured at the level $0.5|\psi|^2_m$.

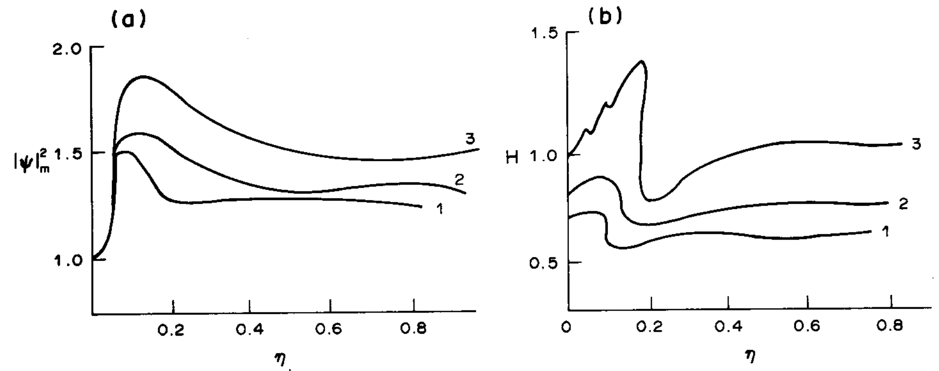

Fig. 2. Effect of the periphery of the pulse (3) on the evolution of the intensity envelope at $\kappa = 4\kappa_{cr}(\nu)$. Curves 1, 2 and 3 correspond to $\nu = 5$, 3 and 1 respectively. The width at half-maximum H is measured at the level $0.5|\psi|^2_m$.

Rather than use (2) we discuss the parabolic finite pulses

$$\psi_0(\tau) = \begin{cases} (1-\tau^2)^{\nu/2}, & \tau^2 \leqslant 1, \\ 0, & \tau^2 \geqslant 1, \end{cases} \tag{3}$$

which carry the dimensionless energy

$$W = \int_{-1}^{1} |\psi_0(\tau)|^2 \, d\tau = \sqrt{\pi}\Gamma(\nu+1)/\Gamma(\nu+\tfrac{3}{2}), \tag{4}$$

where the domain of the gamma function Γ includes a parameter ν.

Figures 1 and 2 illustrate the evolution rate of envelopes with steep ($\nu \leqslant 2$) and smoothed ($\nu > 2$) leading edge. Figure 1 represents the pulse intensity maximum $|\psi|^2_m$ and the corresponding halfwidth H as functions of variable η at $\nu = 2$ for several values of κ. The trends of the halfwidth variation are opposite to that of the intensity maximum. For comparison, the plots corresponding to a linear evolution ($\kappa = 0$) are also given.

For greater values of κ the oscillatory evolution is seen to give way to a monotonic evolution for $\kappa \approx 4\kappa_{cr}$, where

$$\kappa_{cr} = \Gamma^3\left(\frac{\nu+3}{2}\right) \Big/ \Gamma^2\left(\frac{\nu+2}{2}\right). \tag{5}$$

At distances of $\eta \lesssim Z$ the pulse is almost steady. A further increase of κ again intensifies oscillatory trends in the dynamic behaviour of the pulse. Thus, the range of values near $\kappa = 4\kappa_{cr}$ corresponds

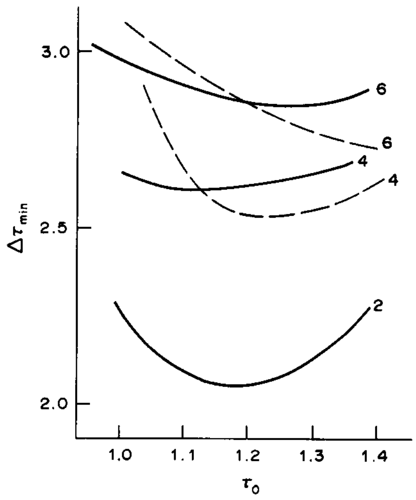

Fig. 3. Minimum admissible spacing between two pulses, $\Delta\tau_{min}$, which after travel over a distance η (shown at the curves) remain resolvable by the Rayleigh criterion ($\kappa = 2$), versus the initial pulse length τ_0.

to a steadiest pulse. Figure 2 illustrates a monotonous evolution of the various envelopes of the family (2) at $\kappa = 4\kappa_{cr}$.

A topical problem in the construction of a communication system is to estimate the maximum admissible data transmission rate. To answer this question we investigated the interaction of two pulses propagating in a single-mode fibre and evaluated the maximum distance of travel at which the pulses were still distinct.The distinguishability was understood in the sense of Rayleigh, i.e. the pulses were deemed resolvable if the irradiance midway between the two pulses dipped below the half-maximum level.

The initial condition for the nonlinear Schrödinger equation (1) was taken in the form

$$\psi_0(\tau) = \psi_0(\tau - \Delta\tau) + \psi_0(\tau + \Delta\tau),$$

where $\Delta\tau$ is the half-spacing between the centres of the two pulses. Two types of envelope were considered, Gaussian $f(\tau) = \exp(-\tau^2/2\tau_T^2)$ and hyperbolic secant $\psi_0(\tau) = \cosh^{-1}(\sqrt{\kappa/2}\cdot\tau)$, and the scaling factor τ_T of the Gaussian pulse was selected so that the half-power widths H of both pulses were equal at the onset of evolution.

The results of the evolution are presented in Fig. 3. The minimum transmission pulse spacings for resolvable detection are given as functions of pulsewidth τ_0 for fixed lengths of transmission lines in units of η. The maximum attainable transmission rate is about $\Delta\tau_{min}^{-1}$. Each length of communication is associated with its own optimal pulsewidth τ_0 of the given envelope. Assuming that the pulsewidth τ_0 is selected appropriately, the maximum transmission rate for Gauss-shaped pulses can be higher than for pulses of hyperbolic secant shape.

Figure 3 shows how two pulses interact until the overlapping intensity satisfies the Rayleigh criterion. After that there may be periodic regimes of coalescence and stratification of the pulses. For solitons, such a regime has been reported by Hasegawa [3]. The same trend is evident in the evolution of parabolic pulses (3) for the same initial spacing $\tau = 4$.

REFERENCES

1. I. N. Sisakyan and A. B. Shvartsburg. *Kvantovaya Elektron.* (Moscow) **11** (9), 1703 (1984).
2. A. B. Shvartsburg and M. A. Zuyev. *Opt. Quant. Electron.* **12**, 95 (1980).
3. A. Hasegawa. *Opt. Quant. Electron.* **14**, 104 (1982).