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SYNTHESIS OF REFLECTING POLARIZERS AS A GRID OF PARALLEL STRIP CONDUCTORS PLUS A PLANE MIRROR

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Abstract—The paper considers the problem of synthesis of reflecting polarizers that transform the polarization of an incident field in a desired manner. The reflecting polarizers are designed as a grid of parallel strip conductors set above a plane mirror. How to evaluate the orientation of the conductors and the dimensions of polarizer elements is described. The results are discussed of the synthesis of reflecting polarizers transforming a linearly polarized wave into a wave with circular polarization. The stability of such polarizers to perturbations of the initial wavelength is investigated.

Evaluating the anisotropy parameters of a curvilinear mirror designed to transform the polarization of an incident wave in a desired manner has been approached by Kinber and co-workers [1, 2]. At each point of the mirror surface the anisotropy has been described by the second-order impedance tensor (or reflection matrix) allowing for a transformation of the two orthogonal components of polarization. The coefficients of the tensor at each point of the mirror have been determined by calculating for a plane incident wave the required transformation of polarization at the anisotropic plane. This approach has determined the direction of anisotropy lines for the mirror and the impedance in the two orthogonal directions.

This paper is devoted to designing anisotropic reflecting planes with a specified impedance. A reflecting polarizer with a given impedance may be constituted by a multilayered grid and a reflecting surface of corrugated or smooth profile [3, 4]. A simple design of plane polarizer is a periodic grid of parallel conductors placed above a plane mirror. The subsequent discussion will be focused on the creation of such polarizers.

It is assumed that the conductors constituting the grid are flat metal bands that may be placed parallel to the mirror (plane grating) or perpendicular to the mirror (knife or edge grating). The period of a grating may involve one or two conductors of different size. For a given angle of incidence, and given polarizations of the incident and reflected waves, the synthesis problem, i.e. to evaluate the direction of conductors in the grating and the size of polarizer elements, is solved by a numerical minimization of a performance criterion (outlined in Section 3) which measures the difference between the desired and resultant polarization of the reflected wave.

The number of parameters used in the minimization depends on the specific type of polarizer. For a single-element plane grating, three parameters have to be determined: grating period p, band width w and polarizer depth d (Fig. 1a) subject to the constraint $w \leqslant p$. In the case of a two-element plane grating, there are five parameters: period p, first band width w_1, second band width w_2, front edge to front edge spacing w_3 and depth d (Fig. 1b), subject to the constraints $w_1 \leqslant w_3$, and $w_3 + w_2 \leqslant p$.

For a one-element knife grating, three parameters are to be determined: period p, strip height h and depth d (Fig. 1c) subject to the constraint $h \leqslant d$. For the case of a two-element knife grating, there are six parameters: period p, depth d, first strip height h_1, second strip height h_2, and horizontal and vertical edge spacings h_3 and w (Fig. 1d) subject to the constraints $w < p$, $h_1 \leqslant d$, $h_3 \geqslant 0$, and $h_3 + h_2 \leqslant d$. In special cases in which the grating lies on the reflecting plane, the parameters to be determined are period p and depth d for a single-element grating (Fig. 1e), and period p, depth d and spacings w and h (Fig. 1f) subject to the constraints $w \leqslant p$, $h \leqslant d$ and $h \geqslant 0$. In all these situations, in addition to the parameters mentioned, the orientation of the grids must be determined.

METHODS OF ANALYSIS OF REFLECTING POLARIZERS

To solve the synthesis problem one needs to compute and minimize the performance criterion. This cannot be done without calculating the field reflected by the polarizer. We therefore begin

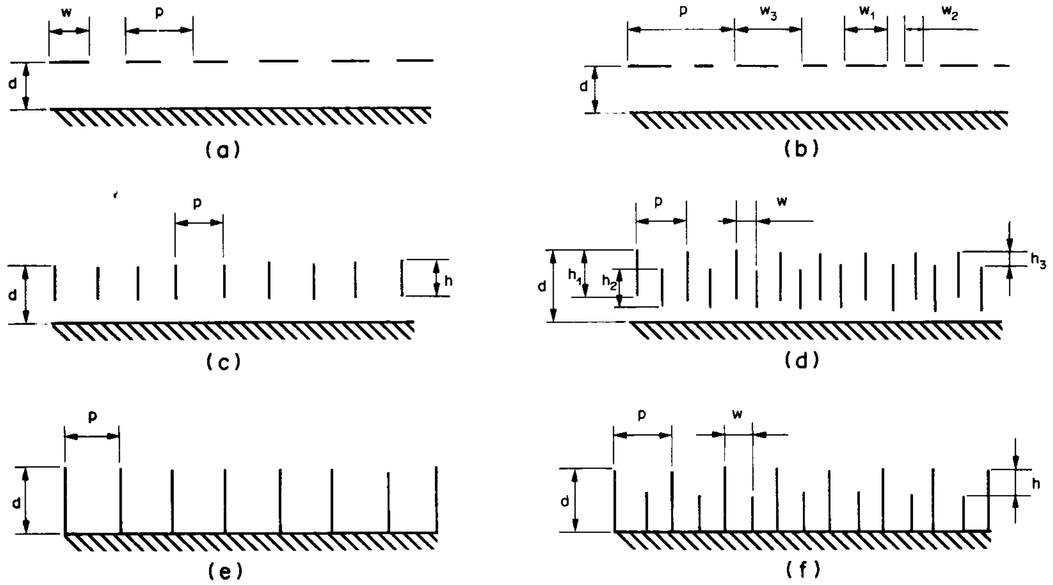

Fig. 1. Reflecting polarizer structures.

with the problem of analysis of the polarizers, which may be tackled by way of analysis of multilayered periodic gratings composed of strip conductors. (The polarizers under discussion represent a particular case of such a structure, because one of the gratings can be composed of horizontal strips equal in width to the grating period.)

One of the methods used for analysing such structures is the Riemann–Hilbert method that reduces the initial problem to solution of a reduced system of linear algebraical equations [5, 6]. However, the efficiency of this method falls sharply when applied to multilayered and multi-element gratings (in our case this is a two-layered grating; the first layer period contains one or two elements while that of the second layer equals the width of a horizontal strip). This limitation prevents the Riemann–Hilbert method from being used in the synthesis problem.

Galishnikova and Ilinsky [7] reduce the analysis of electromagnetic waves on a periodic structure to Fredholm's equation of the first kind which allows a numerical solution by means of self-regularization. However, a direct application of this method is unsuitable for diffraction analysis on a grating composed of parallel strips because the series defining the matrix elements of the associated system of linear equations converge slowly and entail large consumption of computer time.

In what follows we also compute the field reflected by the polarizer by reducing the problem to numerical solution of the integro-differential equation (H-polarization) and Fredholm's integral equation of the first kind (E-polarization) [8]. However, unlike Balishnikova and Ilinsky [7] we invoked Kummer's transformation and achieved a sizeable improvement in convergence of the series (to the same precision the matrix elements were computed 25 times faster). In addition we took into account that the matrix of the system is Toeplitz and used the respective inversion algorithms; this resulted in substantial savings of computer time and allowed higher orders of approximation.

SELECTION OF A PERFORMANCE MEASURE FOR THE SYNTHESIS PROBLEM

In the synthesis of a reflecting polarizer, the performance measure should take into account the amplitude and phase differences between the orthogonal components of the desired and actually obtained polarization of the reflected wave. For example, it may be in the form

$$I = \sum_{i=1}^{2} (\tilde{A}_i - A_i)^2 + (\tilde{\varphi}_i - \varphi_i)^2, \tag{1}$$

where $\tilde{A}_i$, $\tilde{\varphi}_i$ $(i = 1, 2)$ are the amplitudes and phases of the desired polarization vector, and A_i, φ_i the actual components of reflected wave depending on the structure of the polarizer.

For the particular type of polarizer under investigation (with a grating of parallel conductors) the form of expression (1) and its minimization procedure can be appreciably simplified. The minimization of Eq. (1) simplifies since the desired parameters—the angle between the plane of reflection and the direction of the conductors and the dimensions of polarizer elements—can be determined separately, first the orientation of the conductors, then the elements of the polarizer.

The direction of grid conductors coincides with the direction of the anisotropy line. The latter can be determined algebraically by the given angle of incidence of the initial wave and the polarization vectors referring to the initial and reflected waves [1, 2]. The anisotropy line is chosen to be parallel to one of unit vectors of the antidiagonal representation basis of the impedance tensor.† When the orientation of the grid conductors is known, the form of Eq. (1) is amenable to simplification. For this purpose the polarization vectors of the incident and reflected waves should be decomposed into components one of which lies in the plane orthogonal to the conductors, the other being oriented along the vector product of the first component and the wave propagation vector.

In solving the problem of diffraction by a cylinder of an arbitrarily incident plane wave, the above decomposition corresponds to the cases of H-polarization (component of the polarization vector is in the plane perpendicular to the conductors) and E-polarization. The E and H components of a given polarization of the reflected wave are related to the respective components of the initial wave by the reflection matrix based on unit vectors oriented along the grid conductors and at right angles to them. Notice that on this basis the impedance tensor is antidiagonal, hence the reflection matrix will be diagonal [9]. Also, the reflection occurs without losses, therefore the moduli of diagonal elements are all unity. This implies that when the orientation of the grid is found the E and H components of the given polarization of the incident and reflected waves are equal in absolute value for any feasible dimensions of polarizer elements.‡ Thus, for the problem of synthesis, it will suffice to select such dimensions that the phases of the E and H components of the reflected wave coincide with the desired values or, what is the same, to minimize the function

$$I_1 = (\tilde{\varphi}_{2H} - \tilde{\varphi}_{1H} - \varphi_{2H})^2 + (\tilde{\varphi}_{2E} - \tilde{\varphi}_{1E} - \varphi_{2E})^2 \tag{2}$$

where $\tilde{\varphi}_{1H}$, $\tilde{\varphi}_{1E}$, $\tilde{\varphi}_{2H}$ and $\tilde{\varphi}_{2E}$ are the given phases of the E and H components of polarization of the incident and reflected waves, and φ_{2H}, φ_{2E} are the phases of the respective components of the actually reflected wave. The last quantities may be determined by solving the problem of diffraction of a plane wave incident on the polarizer (periodic structure of parallel conductors) in an arbitrary fashion. Because the solution of this problem is related to the diffraction of an obliquely incident plane wave (plane of reflection is orthogonal to the grid conductors), see, e.g. [10], we evaluate φ_{2H} and φ_{2E} by solving the last problem numerically. This is achieved with the aid of the standard computer codes developed on the basis of [8]. The minimization of (2) is carried out by the deformed polyhedral technique [11] which does not require that the partial derivatives of I_1 should be computed and therefore does not introduce additional errors due to numerical differentiation.

In the synthesis of reflecting polarizers that must retain their properties in a certain range of wavelengths, the minimization should be carried out not for the function (2), but rather for the functional

$$I_2 = \int_{\lambda-\Delta\lambda}^{\lambda+\Delta\lambda} (\tilde{\varphi}_{2H} - \tilde{\varphi}_{1H} - \varphi_{2H})^2 + (\tilde{\varphi}_{2E} - \tilde{\varphi}_{1E} - \varphi_{2E})^2] \, d\lambda, \tag{3}$$

where λ is the operating wavelength and $\Delta\lambda$ is an increment specifying the bandwidth.

RESULTS OF CALCULATIONS

We apply the procedure outlined above to the creation of reflecting polarizers that transform a

† It is worth noting that for polarizers with more complicated gratings, such as multilayered with arbitrarily oriented conductors, rectangular cell arrangements and nonparallel conductors, the anisotropy lines do not coincide with the system orientation.

‡ In this work the period of the structure is assumed so small that only the fundamental harmonic is reflected. A more general case may be considered with a few harmonics being propagated by imposing constraints on the moduli of harmonics other than the fundamental.

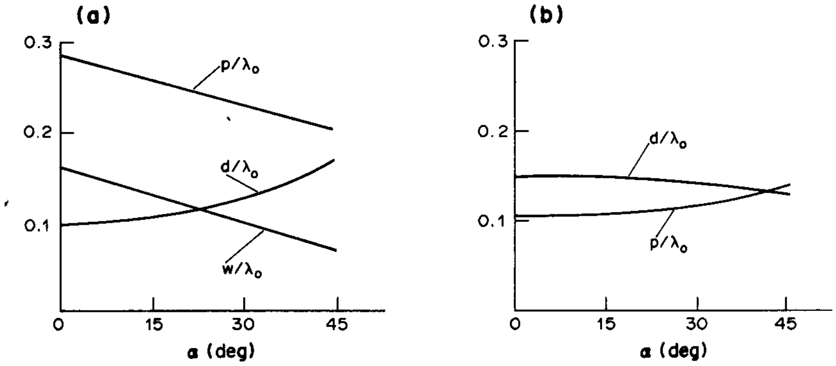

Fig. 2. Relative dimensions of computer-synthesized polarizer elements as functions of incidence angle.

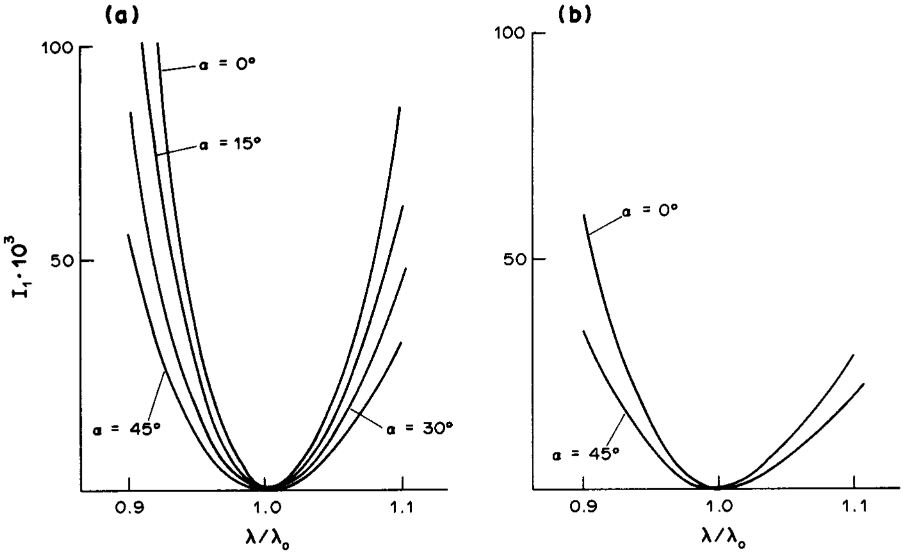

Fig. 3. Objective function I_1 versus wavelength.

linearly polarized wave into a wave with circular polarization. Assume that the electric field strength vector of the incident wave makes an angle of 45° with the plane of reflection. It is not hard to verify that for the given polarizations the direction of the grid conductors will be perpendicular to the plane of reflection. We seek to determine the dimensions of elements of the polarizers containing a single-element plane or knife grating (see Fig. 1a,e) by minimizing the function I_1 given by (2). Let the band thickness be $0.03\lambda_0$ (λ_0 = operating wavelength), $\tilde{\varphi}_{1H} = \tilde{\varphi}_{1E} = 0$, $\tilde{\varphi}_{2H} = 3\pi/2 - \pi/24$ $(-\pi/2 + \pi/24)$, and $\varphi_{2E} = \pi - \pi/24$. (The phase $\tilde{\varphi}_{2E} = \pi$ corresponds to the reflection of the E component of the field from a smooth mirror and may be hard to attain for semitransparent structures. Therefore the phases of both components of the polarization of the reflected field are given with a retardation of $\pi/24$.) The results of these calculations are plotted in Figs 2–4.

Figure 2 shows the dimensions p/λ_0, d/λ_0 and w/λ_0 of the elements of polarizers as functions of angle of incidence (deg.), for the polarizer with a plane grid at (a) and for the edge grid structure at (b). As the angle of incidence d increases, the period p/λ_0 and the band width w/λ_0 of the plane grating decreases, whereas the depth of the polarizer d/λ_0 increases. The parameters of the edge grid polarizer vary more slowly than the plane grid counterpart in the same range of angles α. At higher values of α the period p/λ_0 of the edge grating increases and the depth of the polarizer decreases. Comparison of the diagrams at (a) and (b) reveals that at angles of incidence close to 45° the edge grid polarizer offers a more compact structure because of its smaller depth, d/λ_0.

At small angles α the advantage of smaller size goes to the plane grating structure. However, Fig. 2(b) indicates that at $\alpha \in [0°, 20°]$ the parameters of the edge grid structure remain practically

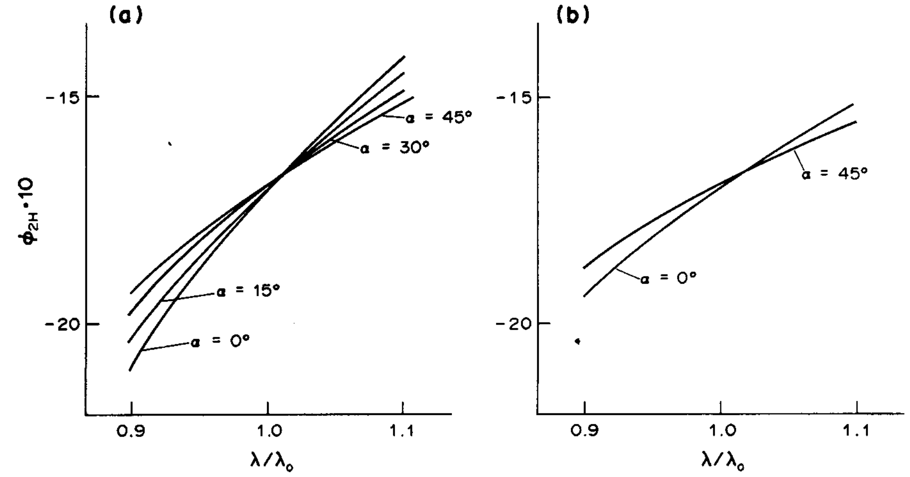

Fig. 4. Sensitivity of the phase φ_{2H} of the field reflected from computer-synthesized polarizers (a, plane grating; b, edge grating) to variations of the operating wavelength.

constant, that is, the performance of this polarizer in this range is independent of the angle of incidence.

The degree of stability of synthesized polarizers with respect to variations of wavelength may be characterized by values of the performance criterion I_1. Figure 3 shows I_1 as a function of wavelength for $\pm 10\%$ detuning from the operating wavelength and four angles of incidence, $\alpha = 0°$, 15°, 30° and 45°. The diagram at (a) relates to the polarizer with a plane grating and the diagram at (b) to the polarizer with an edge grating. The curves for $\alpha = 0°$ and 30° not shown in (b) must lie between the extreme curves for $\alpha = 0°$ and 45°. The function $I_1(\lambda/\lambda_0)$ increases more slowly for increasing wavelengths than for decreasing, and for larger angles α than for smaller. Comparison of the plots in Fig. 3(a,b) indicates that the edge grating polarizer possesses a higher degree of stability with respect to detuning in a wide range of incidence angle α.

To select a design of a polarizer with higher stability the designer should evaluate the contribution of each phase φ_{2E} and φ_{2H} to I_1. Calculations indicate that for the computer-synthesized polarizers with plane and edge gratings, in the ranges of $\alpha \in [0°, 45°]$ and $\lambda/\lambda_0 \in [0.9, 1.1]$, the values of φ_{2E} lie in the interval [2.99, 3.02] (with the desired value 3.01). In other words, the variation of I_1 is almost completely controlled by the variation of φ_{2H}. Figure 4 shows that the interval of variation of φ_{2H} is smaller, i.e. the stability margin is higher, for the edge-grating polarizer at (b) than for the plane grating counterpart at (a).

The plots in Fig. 2 suggest that the problem at hand has a unique solution. However, these polarizer designs are particular cases of more complicated versions (see Fig. 1b,c,d,f), so that a similar synthesis problem for the latter may have indefinitely many solutions. Additional elements of polarizers affect φ_{2H} only insignificantly, and hence the solution of the synthesis problem by minimizing I_2 given by (3) subject to the given polarizations of the incident and reflected waves would not result in a sharp increase of stability margin of the reflecting polarizers.

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